

ISD97, A COMPUTER PROGRAM TO ANALYZE DATA FROM A SERIES OF *IN SITU* MEASUREMENTS ON A GRID AND IDENTIFY POTENTIAL LOCALIZED AREAS OF ELEVATED ACTIVITY

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ABSTRACT

A computer program, ISD97, was developed to analyze data from a series of *in situ* measurements on a grid and identify potential localized areas of elevated activity. The ISD97 code operates using a two-step process. A deconvolution of the data is carried out using the maximum entropy method, and a map of activity on the ground that fits the data within experimental error is generated. This maximum entropy map is then analyzed to determine the locations and magnitudes of potential areas of elevated activity that are consistent with the data. New deconvolutions are then carried out for each potential area of elevated activity identified by the code. Properties of the algorithm are demonstrated using data from actual field measurements.

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INTRODUCTION

An important requirement in surveying for residual radioactivity is the detection of localized areas of elevated contamination, sometimes referred to as hot spots. We have developed a computer program that analyzes data from a series of *in situ* gamma-ray spectrometry measurements on a grid and searches for hot spots that might be “hidden” in the data. The program is designed to be used as a tool when evaluating compliance with regulations that set limits on the size and magnitude of hot spots. Since the program uses *in situ* measurements, the data are nuclide specific and reproducible. Therefore, our method is useful where standard techniques, such as scanning surveys, do not have the necessary sensitivity.

ANALYSIS OF *IN SITU* MEASUREMENTS ON A GRID

In the *in situ* gamma-ray spectrometry technique, a detector typically positioned 1 m above the ground facing downwards is used to measure the spectral distribution of photon fluence due to gamma emitting radionuclides in the soil. The gamma emitting radionuclides can be identified by their specific photon energies, which are registered as peaks in the spectrum. The peak count rate is related to the full absorption of unscattered gammas. If the detector is properly calibrated, the activity per unit mass for a given radionuclide of interest can be derived from the peak count rate using parameters that describe the soil characteristics (such as composition, density, and water content) and the depth profile of the distribution. The reader is referred to Beck et al. (1972), ICRU 53 (1994), Miller and Shebell (1995), and Huffert and Miller (1995) for more information on *in situ* gamma-ray spectrometry.

The *in situ* technique is particularly well suited for quickly determining levels of contamination over large areas. Each measurement provides a weighted average over the detector field of view, which is on the order of many square meters. For example, at a detector height of 1 m, about 30% of the total fluence rate measured comes from beyond a 3-m radius for ^{137}Cs that is uniformly distributed with depth in the soil. For a surface source distribution, about half the fluence comes from beyond 10 m (ICRU 1994). A series of measurements done on a grid on the order of meters apart will, therefore, provide overlapping coverage of the ground area.

A grid spacing close enough for overlapping coverage is required to ensure that any areas of elevated activity between grid points are not missed. For an elevated area to be detected, the minimum excess peak count rate at a detector due to the elevated area must exceed the uncertainty of the measurement. That is, the elevated area has to be of enough intensity to be seen above the level of noise. In this report, it will be assumed that the *in situ* measurements are made close enough so that there is overlapping coverage of the ground area. For most radionuclides of interest, a separation of 5 m is adequate to ensure overlapping coverage.

Although a series of measurements performed on a grid provides valuable information on the spatial distribution of the contaminant, a unique continuous solution cannot be derived from the set of measurements (this observation is not restricted to data collected with the *in situ* technique but holds generally for any set of measurements on a grid, such as soil samples). This is an important point that can be understood intuitively, since determining a continuous distribution from a finite number of measurements without making assumptions about the distribution is clearly not possible. Difficulties will also arise due to experimental errors associated with the set of measurements.

To select one distribution among those that satisfy the data requires additional information regarding the pattern of the deposition of the radioactivity on the ground. In some special cases where the process that causes the deposition of radioactivity on the ground is well understood, making some simplifying assumptions regarding the distribution is possible. One such case is undisturbed fallout from nuclear tests where the deposition is caused by the precipitation of radioactive material from a radioactive cloud of a large size. Here, one expects deposition that is roughly constant over areas that contribute to the detector's count rate (a circle of a few meters in diameter). Using this assumption, selecting a unique distribution of activity on the ground from among the many that fit the data is then possible. However, for many cases of interest, there are no reasonable assumptions that can be made regarding the pattern of activity on the ground. These include areas for which the history of contamination is not well known, and areas where activities such as remediation can affect the level of contamination. Here, the analysis of the data might require considering many possible scenarios that fit the data and checking to see if any such distributions will exceed regulatory limits. The code ISD97 was written to implement this approach. Given a set of *in situ* measurements on a grid, the code ISD97 determines locations and magnitudes for potential areas of elevated activity, and for each it generates a potential distribution of activity on the ground that fit the data. In practical applications, if any of these potential distributions appear to exceed the regulatory limits, further field work (such as soil

sampling, collimated measurements, additional measurements on a closer grid, etc.) would be used to determine whether such an elevated area was present.

D ECONVOLUTION OF INSITU MEASUREMENTS ON A GRID

The results from a set of measurements performed over a grid can be plotted on a map and contours drawn to show variability in the count rate and to delineate elevated areas (an example of such a contour plot is shown in Figure 1). It is often assumed that the variability of activity in the ground follows the pattern of the contour plot of the count rate. This is the simplest assumption that can be made, since one can easily calculate conversion factors from count rate to deposition on the ground (using the assumption of spatial uniformity or constant activity) and then "plug in" those conversion factors to obtain deposition on the ground. In general, the distribution of activity on the ground constructed in this way will not fit the measured data, since the assumption of spatial uniformity obviously does not hold. In addition, since contour plots are constructed by interpolation there is a tendency to generate smooth distributions in this way. Contour plots will not allow for any result that exceeds the highest value measured at a grid point, and will not single out potential elevated areas that might be present between the grid points.

To illustrate the difficulties that can be encountered if one attempts to interpret a contour plot as a map of deposition on the ground, consider the simplified case of three measurements on a line. Figure 2 shows three detectors far apart with little overlap in the fields of view, while Figure 3 shows three closely spaced detectors. The use of conversion factors to calculate deposition on the ground assumes a uniform distribution over the field of view of the detector, but this assumption cannot be satisfied where the field of view of different detectors with different count rates overlap. Between measurement points one has to interpolate between different values of activity (which correspond to the different count rates of neighboring detectors), and this introduces an error. In Figure 2 where there is little overlap, the error due to this interpolation will be small, and the contour plot will provide a map of smoothed activity on the ground that fits the data. However, this is a bad setup for a survey because good overlapping coverage of the ground area is essential to assure that elevated areas that fall "between" the grid points are not missed. If we insist on having complete coverage of the ground, we must consider the case illustrated in Figure 3. Here, since there is substantial overlap in the field of view of the detectors, the error due to interpolation will be large. In summary, when the grid of measurements is made

small enough to provide good coverage, a nonuniform distribution of activity on the ground that follows the contour plot will not fit the data.

To analyze data such as those provided in Figure 1 requires more than just preparing a contour plot. It requires carrying out a deconvolution of the data. In deconvolution, one uses the data together with a set of additional conditions to find a distribution of activity on the ground that fits the data and satisfies the given set of conditions. These additional conditions are needed because, as stated before, reconstructing the real distribution of activity on the ground from a finite number of measurements is an ill posed problem, and without additional requirements the solution is not unique. For a given distribution of activity on the ground $f(x)$, the peak count rate N_k measured at location k results from integrating the detector's response $R_k(x)$ over the $f(x)$, and is given by

$$N_k + \epsilon_k = \int R_k(x)f(x)dx \quad (1)$$

where ϵ_k is the (unknown) measurement error, the difference between the true value and the measured value. Deconvolution of the data amounts to finding a distribution $f(x)$ of activity on the ground that fits the data (Equation 1 for all measurements) and satisfies the given set of conditions. These conditions can be mathematical (e.g., one can choose the solution to be the smoothest function that satisfies the data) or these conditions may be derived from physical considerations (e.g., one can choose the solution that is closest to a given model while still satisfying the data). An important requirement in surveying for residual radioactivity is the detection of localized areas of elevated contamination, and one possible condition that can be imposed is to require that the solutions include the largest localized areas of elevated activity suggested by the data. This is of interest since regulations limit the magnitude and size of such areas, and given a set of data one would like to know if it is possible that a localized area of elevated activity might be “hidden” within that data. The computer code ISD97 unfolds the *in situ* data with the use of such a condition.

THE CODE ISD97

The computer code ISD97 is a FORTRAN 90 program that generates solutions to the deconvolution problem using a two-step process. The methodology is described in detail below, but for those already familiar with the maximum entropy method it can be summarized as follows. In the first part of the code, a deconvolution of the data is carried out using the maximum entropy principle with the assumption of a constant default (background) level. The output of the maximum entropy deconvolution is an activity distribution that tends to be flat almost everywhere except in the neighborhood of each measurement point, where there will be a localized peak or a depression depending on how the measured peak count rate at that measurement point compares with the average peak count rate. This makes it possible to associate with each measurement a given level of activity in the ground that can be used to identify the location and magnitudes of potential areas of elevated activity. In the second part of the code, the maximum entropy solution is analyzed, and this information is used to find the locations and magnitudes of potential areas of elevated activity. New deconvolutions are then carried out for each potential area of elevated activity. This second series of deconvolutions are done using the maximum entropy principle with default levels set equal to a constant (background) level, plus an area of elevated activity superimposed on it. A diagram summarizing the algorithm is shown in Figure 4, and a complete listing of the code is in Appendix B.

To formulate the deconvolution problem in discrete terms, we divide the distribution of activity on the ground into a large number of equal-sized cells, the i^{th} cell having activity f_i . In this way, the continuous distribution of activity on the ground $f(x)$ is approximated by point sources f_i at each cell i . Throughout this report we use the following notation. The peak count rate measured at point k is N_k and σ_k is the statistical counting error at 1σ . R_{ki} is the matrix representing the instrument response (the count rate measured at point k per unit activity in cell i) and $N_k = \sum R_{ki} f_i$ is the calculated peak count rate at the measurement point k for a given activity distribution f_i . We use the notation f_i^{def} to denote the default level of the cell i (default levels have to be specified for the maximum entropy deconvolution of the data). Finally, we use the expression “fit the data” to mean that the chi-square,

$$\chi^2 = \sum \epsilon_k^2 / \sigma_k^2 \quad (2)$$

is on the order of the number of measurements (and, therefore, the distribution of activity folded with the detector response will have for each measurement an error equal on average to the uncertainty of the measurements).

The optimal cell size is determined from the following considerations: the cells must be small enough to ensure that such an approximation does not introduce a substantial error in the calculation, but large enough to reduce the total number of cells needed to model the ground and in this way reduce the computational burden. Square cells with sides measuring 50 cm have proven adequate to balance these two conflicting needs. The potential areas of elevated activity are modeled as squares made from N^2 cells (where N is an integer, $N=1,2, \dots$). Their size is chosen by the user. This choice will depend, in general, on the regulations that are applicable to the survey area.

All the information needed for the deconvolution of the data is read in at the beginning, either by the main program or by the subroutine `ISD`. The code requires an input file that defines:

- (a) the number of cells used to model the ground, which is modeled as a rectangle, and the size of each cell (in cm),
- (b) the number of measurements, and for each measurement k its coordinates x_k, y_k (in cm),
- (c) the peak count rate N_k and associated uncertainty σ_k (in counts min^{-1}) for each measurement,
- (d) the photon energy being measured (in keV),
- (e) the detector ID code (the detector response is calculated by the subroutine `RESPONSE`, which includes the detector parameters), and
- (f) an internal parameter used to establish a length scale for the problem (used in the subroutine `PEAKXELS`).

In addition, the user is prompted to enter the names of the input and output files, and the size of the potential areas of elevated activity. Finally, the user is given a choice for modeling the depth profile of the activity on the ground (uniform distribution or surface distribution), a choice of count rates to be used to set the background activity, and a choice of units to be used to report the final activity.

Since the potential areas of elevated activity have to be superimposed over a background activity, a choice of background activity needs to be made. This background activity can be estimated from either the data or it can be entered if known *a priori*. The uniform background distribution is calculated from the user's choice of one of five count rate values:

- (a) the average count from all measurements,
- (b) the median value of the count rates,
- (c) the count rate from the lowest measurement,
- (d) a weighted average that minimizes the chi-square of the background distribution, and
- (e) a user determined count rate.

Choice (e) is recommended if the count rate due to the background activity can be estimated *a priori*, for example, from a neighboring reference area, while (d) is recommended if no information regarding the background distribution is available. The other choices made available to the user can be useful under special circumstances. Selecting (d) sets the background activity for each cell equal to

$$f_i \equiv (\sum N_k(\sum R_{ki})/\sigma_k^2) / (\sum (\sum R_{ki})^2/\sigma_k^2). \quad (3)$$

After the initial distribution is chosen, the next step is to carry out the maximum entropy deconvolution of the data. This is done by the subroutine MAXENT, with the f_i^{def} set equal to the background level. This completes the first part of the algorithm. The results of this first part is the determination of a background activity level (which normally will not fit the data) and a maximum entropy deconvolution of the data that fits the data. As we pointed out before, the maximum entropy solution tends to be flat almost everywhere except in the neighborhood of each measurement point, where there will be a localized peak or a depression depending on how the measured peak count rate at that measurement point compares with the average peak count rate.

In the second part of the code, the maximum entropy solution is analyzed to determine the positions and intensities of potential localized areas of elevated activity that might be present in

the data. The subroutine PEAKXELS associates an integrated activity (IA) with each measurement by adding the activity that lies within a circle centered about the measurement point and subtracting from it the background activity. The circles are made as large as possible without having them overlap. The IAs will be positive or negative depending on whether there is an excess or a deficit of activity associated with that measurement point. We point out that making a quantitative correspondence between measurement points and localized activity using the peak count rates is not trivial due to the overlapping fields of view of the detectors and the nonuniformity of their response functions (which close to the measurement point varies roughly as the inverse distance squared). The assumption is made that any potential areas of elevated activity will be found in the neighborhood of those measurement points where the IA is positive. For each potential area of elevated activity, the code carries out a full analysis (as described below) that includes in each case a new deconvolution of the data.

After determining that a measurement point has a positive IA, the subroutine FINDHS is used to find the location of the area of elevated activity associated with that measurement point. This is achieved by comparing the IA at that measurement point with the IAs of the neighboring measurement points (for example, for a square grid the code compares the IA of the measurement point in question with those of the eight measurement points closest to it). FINDHS uses the inverse distance squared relationship and finds the optimal location for a point source by choosing a location that minimizes the error incurred when the activity in the IAs of neighboring measuring points is replaced by the point source (the procedure is described in more detail in the section on the subroutine FINDHS). After the positions of the potential areas of elevated activity are fixed, the magnitudes are determined by the subroutines MAGPS and MAGPSNN. The magnitude chosen is the largest one that minimizes either the chi-square of the measurements in the immediate neighborhood of the point source, or the chi-square of the measurement that is closest to the area of elevated activity. Finally, new deconvolutions are done taking into account the information on the position and magnitudes of the potential areas of elevated activity. For these final deconvolutions, the f_i^{def} are set equal to the background level plus the area of elevated activity superimposed on it. Except for the way in which f_i^{def} are chosen, the deconvolutions proceed using the maximum entropy method as described before.

The output consists of four files, one file that includes tables of results and three files that contain maps of activity distributions for plotting. The first file, which has extension "TBL," includes:

- (a) the name of the file,
- (b) the detector ID code and photon energy,
- (c) the average count rate and the value of the uniform activity associated with that average count rate,
- (d) the background count rate and the value of the uniform activity associated with that background count rate,
- (e) the chi-square of the uniform background activity distribution,
- (f) the chi-square of the maximum entropy solution,
- (g) parameters related to the largest potential elevated area, such as the location, magnitude, area, and the chi-square of the distribution associated with this elevated area,
- (h) a table with the measured counts, calculated counts and the ratio of measured to calculated peak count rates at each measurement point for the activity distribution of the largest potential area of elevated activity, and
- (i) a table with the coordinates and magnitudes of all potential areas of elevated activity, and the chi-square of the distribution associated with each elevated area.

The second file, which has extension “MXE,” is the output from the maximum entropy deconvolution. The third file (which has a blank extension) is the activity distribution corresponding to the largest potential area of elevated activity over a smooth background. The fourth file, which has extension “ALL,” is a collection of all the potential areas of elevated activity plotted over a smooth background.

ALGORITHMS

A description of the algorithms used in the most important subroutines follows.

THE SUBROUTINE MAXENT

We now present the maximum entropy deconvolution algorithm used in the code, which is a modification of the one in Wilczek and Drapatz (1985). For a discussion of the maximum entropy principle, we refer the reader to Appendix A, and to the references contained therein.

Using the notation described at the previous section, we define the set of distributions that fit the data using two restrictions:

$$N_k + \epsilon_k = \sum R_{ki} f_i, \quad (4)$$

$$\sum \epsilon_k^2 / \sigma_k^2 = \Omega, \quad (5)$$

where Ω is a parameter that equals the value of the χ^2 of the solution (typically, Ω is set equal to the number of measurements), k labels the measurement ($k=1, \dots, m$) and i labels the cell ($i=1, \dots, n$). The set of equations (4) is the discretized version of (1). Equation (5) is a χ^2 -constraint for handling the (unknown) errors ϵ_k and assumes that the errors are normally distributed with zero mean and variances σ_k^2 . From this set of admissible distributions we want to select the one that maximizes the entropy S of the distribution,

$$S = - \sum \{f_i \log (f_i / f_i^{\text{def}}) + f_i^{\text{def}} - f_i\} \quad (6)$$

where the f_i^{def} is the (discretized) default distribution. Equation (6) is in the form given in Skilling (1989).

The Lagrangian associated with the maximization of (6) with constraints (4) and (5) is of the form

$$L(f_i, \epsilon_k, \lambda_k, \mu) = - \sum \{f_i \log(f_i / f_i^{\text{def}}) + f_i^{\text{def}} - f_i\} - \sum \lambda_k \{ \sum R_{ki} f_i - N_k - \epsilon_k \} - \mu \{ \sum (\epsilon_k / \sigma_k)^2 - \Omega \}. \quad (7)$$

where λ_k, μ are $(m+1)$ Lagrange multipliers. Variation with respect to f_i, ϵ_k and μ lead to the set of $(n+m+1)$ equations

$$- \log(f_i / f_i^{\text{def}}) - \sum \lambda_k R_{ki} = 0 \quad (8)$$

$$\lambda_k - 2\mu \epsilon_k / \sigma_k^2 = 0 \quad (9)$$

$$\sum \epsilon_k^2 / \sigma_k^2 = \Omega. \quad (10)$$

We can use (8), (9) and (10) to solve for the f_i , the ϵ_k and μ in terms of the λ_k ,

$$f_i = f_i^{\text{def}} \exp\{\sum \lambda_k R_{ki}\} \quad (11)$$

$$\epsilon_k = \lambda_k \sigma_k^2 / 2\mu \quad (12)$$

$$\mu = (\sum (\lambda_k \sigma_k)^2 / 4\Omega)^{1/2}. \quad (13)$$

Variation with respect to the λ_k leads to (4), and using (11), (12), and (13) we can write these m equations as

$$N_k + \lambda_k \sigma_k^2 (\Omega / \sum (\lambda_j \sigma_j)^2)^{1/2} - \sum R_{kj} f_j^{\text{def}} \exp\{\sum \lambda_l R_{lj}\} = 0 \quad (14)$$

Thus, the initial optimization problem stated has been reduced to a system of m equations with m unknowns $\lambda_1, \dots, \lambda_m$. These equations can also be derived by maximizing a potential function Z ,

$$Z = - \sum f_i^{\text{def}} \exp\{-\sum \lambda_k R_{ki}\} - (\Omega \sum (\lambda_k \sigma_k)^2)^{1/2} - \sum N_k \lambda_k \quad (15)$$

with respect to the λ_k . Therefore, we can reformulate the problem in terms of the maximization of the potential function Z .

The f_i^{def} are default levels for each cell i . Without the chi-square constraint, the maximum entropy solution would default to $f_i = f_i^{\text{def}}$. When the subroutine MAXENT is called the first time, the f_i^{def} are set equal to a constant, background activity distribution. When it is called for the final deconvolutions (after the initial magnitude of each of the potential areas of elevated activities are determined by the subroutines MAGPS and MAGPSNN), the f_i^{def} are set equal to a background level plus an area of elevated activity superimposed on it.

To find the extremum of the potential function Z , we use the subroutines FRPRMN and LMBDAS. FRPRMN is more robust while LMBDAS converges more quickly. For the deconvolution that gives us the maximum entropy map, we first use FRPRMN with the initial value of the λ_k set equal to zero, and then restart the maximization using LMBDAS. For all other deconvolutions, we use LMBDAS with the initial value of the λ_k set equal to the values that correspond to the maximum entropy solution. If LMBDAS stops converging the deconvolution proceeds using FRPRMN.

THE SUBROUTINE LMBDAS

This subroutine uses an iterative procedure based on the integral equations (14) from which follow the set of equations

$$\lambda_k^{M+1} = (1-p) \lambda_k^M + p (\sum R_{kj} f_j^{\text{def}} \exp\{\sum \lambda_l^M R_{lj}\} - N_k) (\sum (\lambda_j^M \sigma_j)^2 / \Omega)^{1/2} / \sigma_k^2 \quad (16)$$

where M denotes the Mth iterate. The parameter p is a smoothing parameter introduced to help convergence. It is set initially to 0.01 and allowed to increase to up to 0.1 in successive iterations. This iterative procedure requires initial values λ_j^0 satisfying

$$\sum(\lambda_j^0 \sigma_j)^2 \neq 0. \quad (17)$$

THE SUBROUTINE FRPRMN

This subroutine is taken from Press et al. (1992). Given a set of initial values for the λ_k , it uses the conjugate gradient method in multidimensions to maximize the potential function Z.

THE SUBROUTINE PEAKZELS

This subroutine calculates the excess or deficit of activity associated with each measurement position. The IA associated with each measurement is calculated by adding the activity that lies within a circle centered about the measurement point and subtracting from it the background activity f_i^{BKG} ,

$$IA_k = \sum \{f_i - f_i^{BKG}\} \quad (18)$$

where the sum is over the f_i within the circle, centered about the measurement position k. The circles are made as large as possible without having them overlap. The IAs will be positive or negative depending on whether there is an excess or a deficit of activity associated with that measurement point.

THE SUBROUTINE FINDHS

This subroutine calculates the optimal position of the potential area of elevated activity. To find this position, we use the following reasoning. If measurements at neighboring measurement points are in fact due to a single localized area of elevated activity, then it should be possible to replace the IAs of these measurement points with excess activity at a single location and still have a distribution that fits the data.

To carry out this procedure, the potential localized area of elevated activity is approximated by a point source. For each neighboring measurement point k, and for a point source of activity A_e located at an arbitrary point e, we now define an error term δ_{ek} using the equation

$$\sum (IA_j/r_{jk}^2) = (A_e + \delta_{ek})/r_{ek}^2 \quad (19)$$

where IA_j is the integrated activity of measurement point j, r_{ek} is the distance between the cells e and k, and the sum is over the measurement points close to the area of elevated activity. Here, we have taken into account the fact that the detector response for a point source can be approximated by the inverse distance squared relationship. For a given location e, the activity A_e is found by minimizing the magnitude of the error term vector E_e associated with the point e, which we define as

$$E_e = (\sum \delta_{ek}^2)^{1/2}. \quad (20)$$

By systematically calculating E_e for different locations e close to k, one can determine the location at which it is a minimum, and in this way define the optimal position of the potential area of elevated activity. While this procedure involves introducing approximations and it is, therefore, not exact, it has been found to be reliable and to lead to good results.

THE SUBROUTINE MAGPS

This subroutine calculates the optimal intensity of the potential area of elevated activity. The area of elevated activity is modeled with a point source at cell e superimposed on a flat background, and the activity of the point source is calculated by minimizing the chi-square of the measurements in the immediate neighborhood of the point source.

THE SUBROUTINE MAGPSNN

This subroutine provides an alternative calculation of the optimal intensity of the point source used in the modeling of a potential area of elevated activity. The area of elevated activity is modeled with a point source at cell e superimposed on a flat background, and the activity of the

point source is calculated by minimizing the chi-square for the measurement that is nearest to the point source only. The value of the activity calculated by MAGPSNN is used only when it exceeds the activity calculated by MAGPS.

THE SUBROUTINE RESPONSE

This subroutine contains all the relevant information on the instrument response (the count rate per unit activity) for the detectors used, and calculates the response matrix R_{ki} . Currently, the calculation of the response matrix takes into account the inverse distance squared relationship, the photon energy, the photon attenuation in air and ground, detector parameters related to the efficiency and angular response, and a choice of either a uniform or a surface source distribution to model the distribution in depth. This subroutine is in a separate file, and can be easily modified and updated.

TEST CASES

The computer code ISD97 has been tested using both simulated data and actual measurements. In this section we summarize results obtained with data from two different field measurements.

TEST CASE NUMBER ONE

We summarize the results of an analysis of a series of field measurements where a 28.23 kBq ^{137}Cs check source was used to simulate an area of elevated activity. The results described here differ from the analysis in Reginatto et al. (1996) due to a number of improvements that have since been incorporated into the code. In particular, the maximum entropy algorithm and the subroutines that calculate the position and magnitude of the potential areas of elevated activity have been rewritten, resulting in better agreement between the parameters calculated by the code and the actual position and magnitude of the check source used to simulate the area of elevated activity.

The field site chosen was nonuniform, including some areas that had been resurfaced. The ^{137}Cs background distribution varied between close to zero and its expected global fallout level in this area (2.96 kBq m^{-2}). Measurements were made with a 40% relative efficiency germanium

detector placed at a height of 1 m, with a counting time of 10 min at each measurement point. Thirty-five equally spaced measurements of the 662 keV γ fluence were taken on a rectangular 7 \times 5 grid. The measurement points were 2.44 m (8 ft) apart. The statistical uncertainty was $< 5\%$ (1σ) for each measurement. The peak count rate at different measurement points varied by up to a factor of eight due to the variability of the background and to some extent also to the presence of the area of elevated activity. The data was processed in two different ways. In one case, all 35 measurements were taken into account. In the other, every other measurement was discarded, resulting in a total of 12 measurements on a 4 \times 3 rectangular grid with a spacing of 4.88 m (16 ft). For the deconvolution, the area of elevated activity was modeled as one cell, a square 0.4 m \times 0.4 m in size.

Figure 1 shows the peak count rate measured at each measurement point in the grid, along with a contour plot constructed from the data by interpolation. The position of the check source is also shown. It is common practice to assume that the activity on the ground follows a similar contour pattern. As pointed out before, this approximation is not valid when there is good overlap of the field of view of the detectors. For our test case, this is certainly not so, and the contour plot does not provide a good map of the underlying activity distribution that will fit the data. Figure 5 shows the output from the maximum entropy deconvolution, while Figure 6 is the activity distribution that corresponds to the largest single potential area of elevated activity over a smooth background for the 35 measurements case. Both types of deconvolutions generate good solutions in the sense that in both cases the activity distributions fit the data. The maximum entropy solution shows structure that reflects the grid geometry with peaks centered exactly at measurement points with an excess of activity. The solution with the largest area of elevated activity on the other hand has one large peak that falls between measurement points superimposed on a smooth background.

The code located the potential area of elevated activity very precisely, within 22 cm of the actual location of the check source when using 35 measurements, and within 37 cm when using 12 measurements (the code also provided locations for other potential areas of elevated activity that turned out to be related to areas that had higher levels of activity due to variations in background activity, but we will restrict this discussion to the potential area of elevated activity that corresponds to the check source). The code was also very successful in estimating the magnitude of the point source. The calculated magnitude of the point source was 26.93 kBq (within 5% of the true value) when using 35 measurements, and 75.84 kBq (2.7 times the true value) when using 12 measurements. The 12 measurements case is less constrained, and, therefore, allows for a larger potential area of elevated activity.

TEST CASE NUMBER TWO

We summarize an analysis of *in-situ* measurements in a room that had ^{137}Cs contamination on the floor. The floor of the room was rectangular and measured $5.0\text{ m} \times 6.0\text{ m}$. Four measurements at 662 keV γ fluence with a counting time of 10 min were taken with a 40% relative efficiency germanium detector placed at a height of 1 m. To set up the grid, the floor was divided into four quadrants with a measurement centered in each. The statistical uncertainty was on the order of 5% for each measurement, and the peak count rate at different measurement points varied by up to a factor of three due to the presence of the contaminant. In addition to these measurements, 22 collimated measurements taken with the detector placed close to the ground and using a collimator that restricted the field of view to a circle of 35 cm in radius. These collimated measurements were not analyzed with the code ISD97, but were used instead to construct a map of localized activity that can be compared with the results of the deconvolution of the four uncollimated measurements. For the deconvolution, the area of elevated activity was modeled as a square $1.0\text{ m} \times 1.0\text{ m}$ in size. The background level (which was essentially zero) was set equal to the statistical uncertainty. Further details can be found in Miller et al. (1997).

Figure 7 shows a map of the floor with peak count rates plotted for both uncollimated and collimated measurements, and the location of the four uncollimated measurements. In addition, the location of the potential elevated area identified by the code is shown. Although the contamination appears to extend over an irregular area of more than one square meter, the location of the area of elevated activity identified by the code corresponds with good precision to the contaminated area. The code was also very successful in estimating the magnitude of the elevated area. The activity calculated by the code from the four uncollimated measurements equals 0.41 Bq, while the activity derived from the four highest collimated measurements in the neighborhood of the elevated area equals 0.48 Bq (the total inventory calculated from all measurements equals 0.52 Bq).

The data from a few uncollimated measurements, when analyzed with the code ISD97, can provide information that would otherwise require many collimated measurements. Use of this methodology can, therefore, result in substantial savings in time and expense.

CONCLUSIONS

In this report we describe in detail the code ISD97, a computer program that analyzes data from a series of *in situ* measurements on a grid and identifies potential areas of elevated activity. Due to the nonuniqueness of the deconvolution problem, we do not search for a single solution that fits the data, but have developed instead a methodology in which we search for many distributions of activity on the ground, all of which fit the data and have a potential area of elevated activity. This approach seems to be the only one adequate for areas for which the history of contamination is not well known, as well as for areas where activities such as remediation can affect contamination in unforeseeable ways, since then there are no reasonable assumptions that will select one solution from all the possible solutions that fit the data.

The methodology presented here provides an alternative to the standard methods of detecting areas of elevated activity. Of the methods currently in use, one of the most common is a “walkover” using a survey meter gross count rate to signal the presence of an area of high radioactivity. These surveys depend to some extent on the expertise of the technician doing the “walkover” and (unlike regulations) do not discriminate between different radionuclides. The results of such surveys are, therefore, more qualitative than quantitative. The method implemented in the code ISD97 uses *in situ* data taken under controlled conditions. It provides the location of potential areas of elevated activity consistent with the data, and it lists activity levels for all the potential areas of elevated activity found. In this way a quantitative record of the magnitude of areas of elevated activity that might be “hidden” in the data is generated. Analyses of data from actual field measurements (two of which are summarized in this report) show that the methodology is directly applicable to practical cases.

REFERENCES

Beck, H. L., J. De Campo and C. K. Gogolak

In Situ Ge(Li) and NaI(Tl) Gamma-Ray Spectrometry

U.S. Atomic Energy Commission, New York, Report HASL-258 (1972)

Huffert, A. M. and K. M. Miller

Measurement Methods for Radiological Surveys in Support of New Decommissioning Criteria

NUREG-1506 (Draft Report for Comment), Washington, D.C.: Nuclear Regulatory Commission (1995)

International Commission on Radiation Units and Measurements

Gamma-Ray Spectrometry in the Environment

Bethesda, MD: ICRU Publication 53 (1994)

Jaynes, E. T.

Information Theory and Statistical Mechanics

Phys. Rev. 106:620-630 (1957)

Jaynes, E. T.

Prior Probabilities

IEEE Trans. Syst. Cybern. SSC-4:227-241 (1968)

Miller, K. M. and P. Shebell

In Situ Gamma-Ray Spectrometry - A Tutorial for Environmental Radiation Scientists

USDOE Report EML-557 (1995)

Miller, K. M., M. Reginatto, P. Shebell, G. A. Klemic, and C. V. Gogolak

Spectrometric Techniques for the Measurement of Radioactive Contamination in Buildings

Proceedings Air and Waste Management Association's 90th Annual Meeting and Exhibition

Toronto, Ontario, Canada, June 8-13 (1997)

Press, W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannery

Numerical Recipes in FORTRAN: the Art of Scientific Computing

Cambridge University Press, New York, NY, Second Edition (1992)

Reginatto, M., P. Shebell and K. M. Miller

An Application of the Maximum Entropy Method for Assessments of Residual Radioactivity
at Contaminated Sites

IEEE Trans. Nucl. Sci. 43:1837-1841 (1996)

Shannon, C. E.

A Mathematical Theory of Communication

Bell Syst. Tech. J. 27:379-423, 623-656 (1948)

Shore, J. E. and R. W. Johnson

Axiomatic Derivation of the Principle of Maximum Entropy and the Principle
of Minimum Cross-Entropy

IEEE Trans. Info. Theory IT-26: 26-39 (1980)

Skilling, J.

Classic Maximum Entropy

In: J. Skilling (Editor), Maximum Entropy and Bayesian Methods

Kluwer Academic Publishers, Dordrecht (1989)

Wilczek, R. and S. Drapatz

A High Accuracy Algorithm for Maximum Entropy Image Restoration
in the Case of Small Data Sets

Astron. Astrophys. 142: 9-12 (1985)

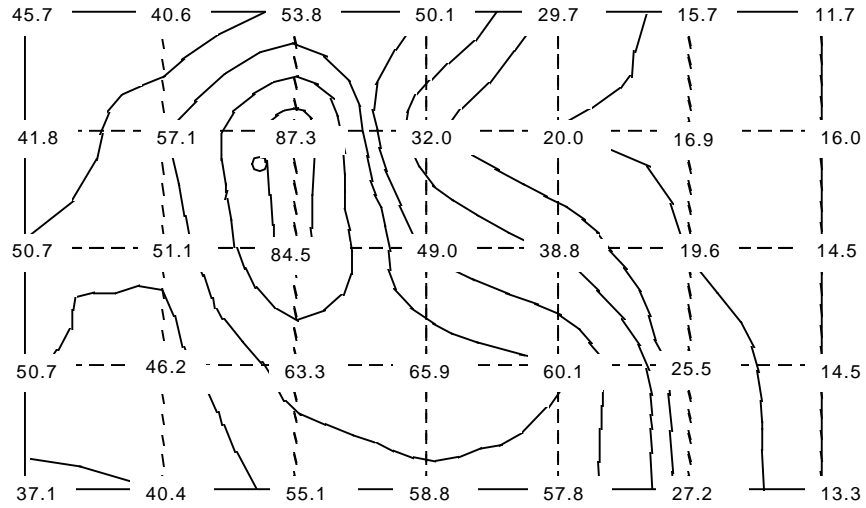


Figure 1. Grid with peak count rates (counts min⁻¹) at each measurement point. The circle indicates the placement of a check source used to simulate an elevated area.

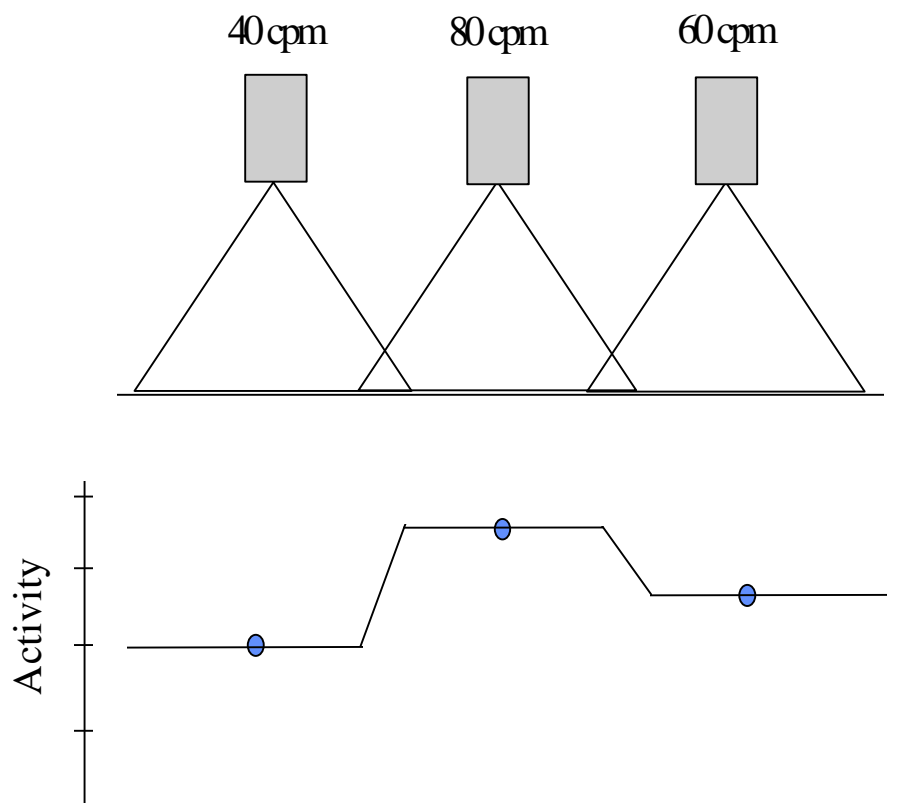


Figure 2. Map of activity on the ground (constructed from the count rate contour plot) for three detectors with little overlap.

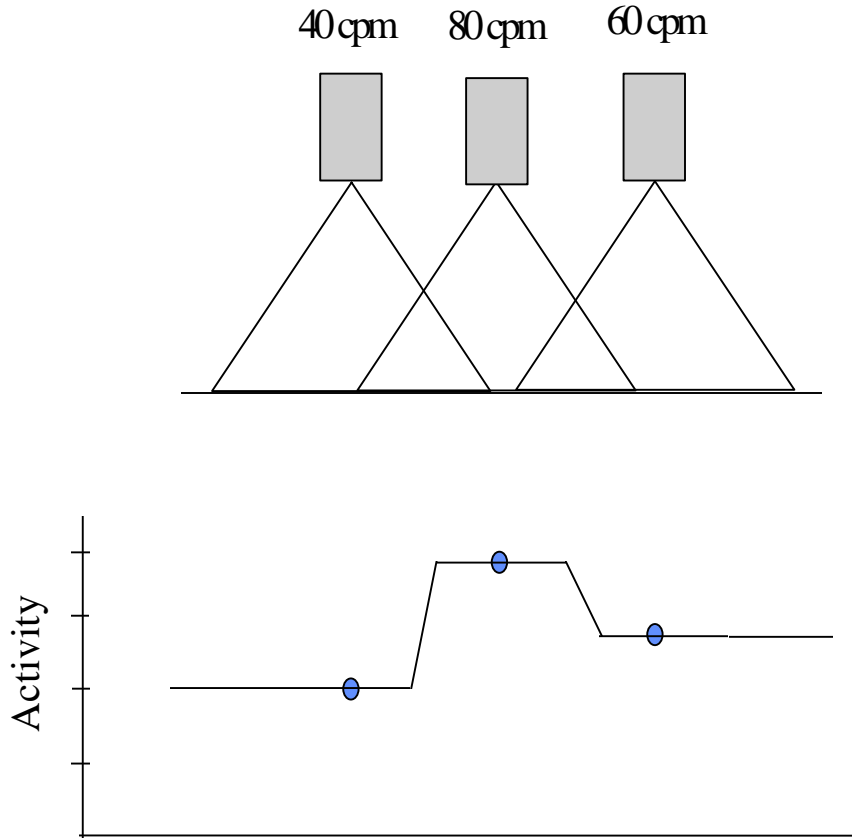


Figure 3. Map of activity on the ground (constructed from the count rate contour plot) for three closely spaced detectors.

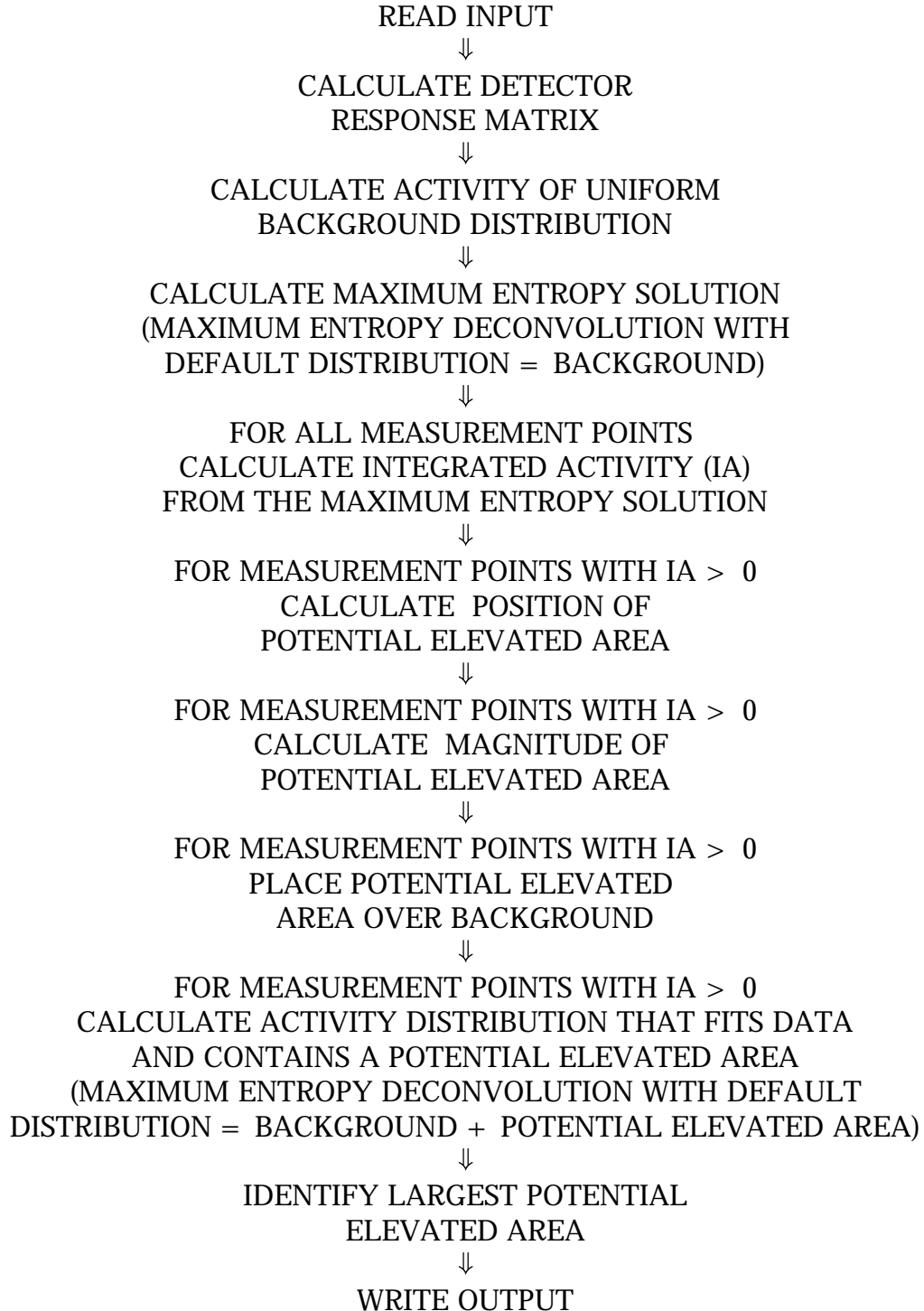


Figure 4. Algorithm of code ISD97.

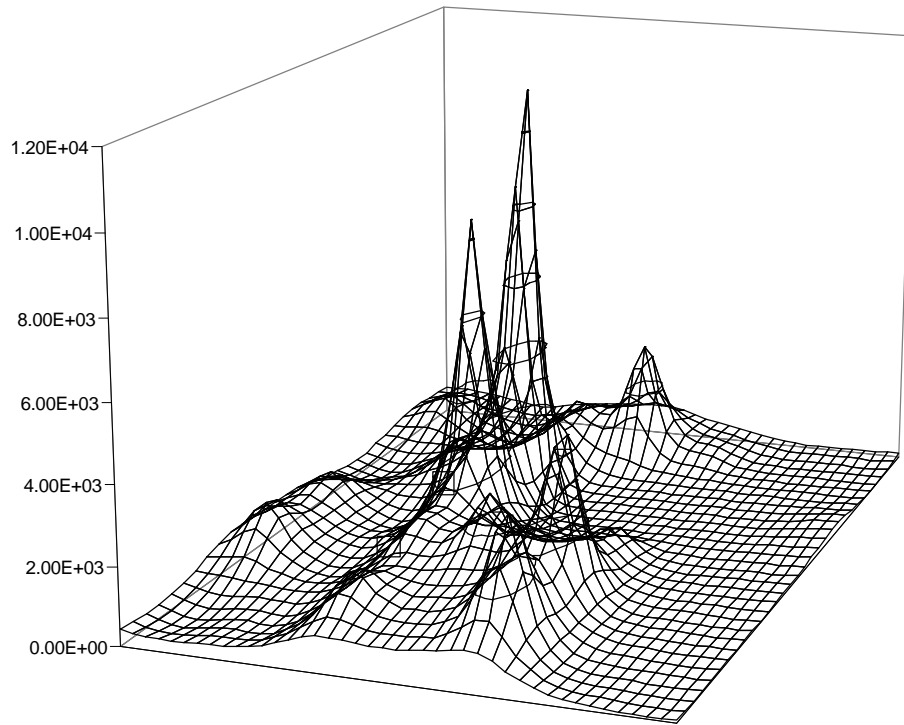


Figure 5. Activity distribution (Bq m^{-2}) from maximum entropy deconvolution (test case 1).

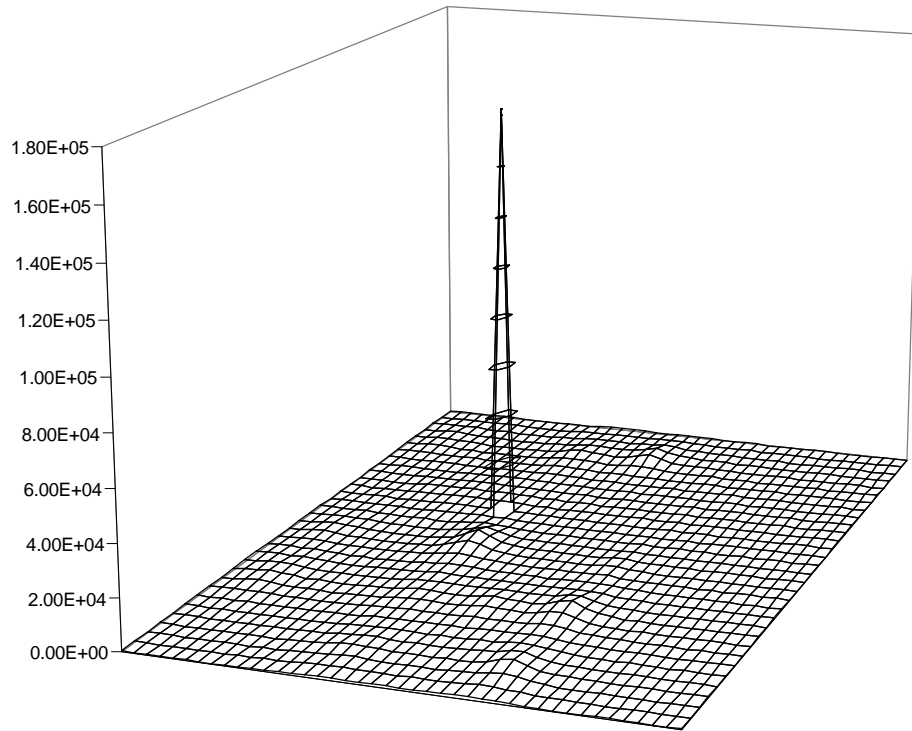
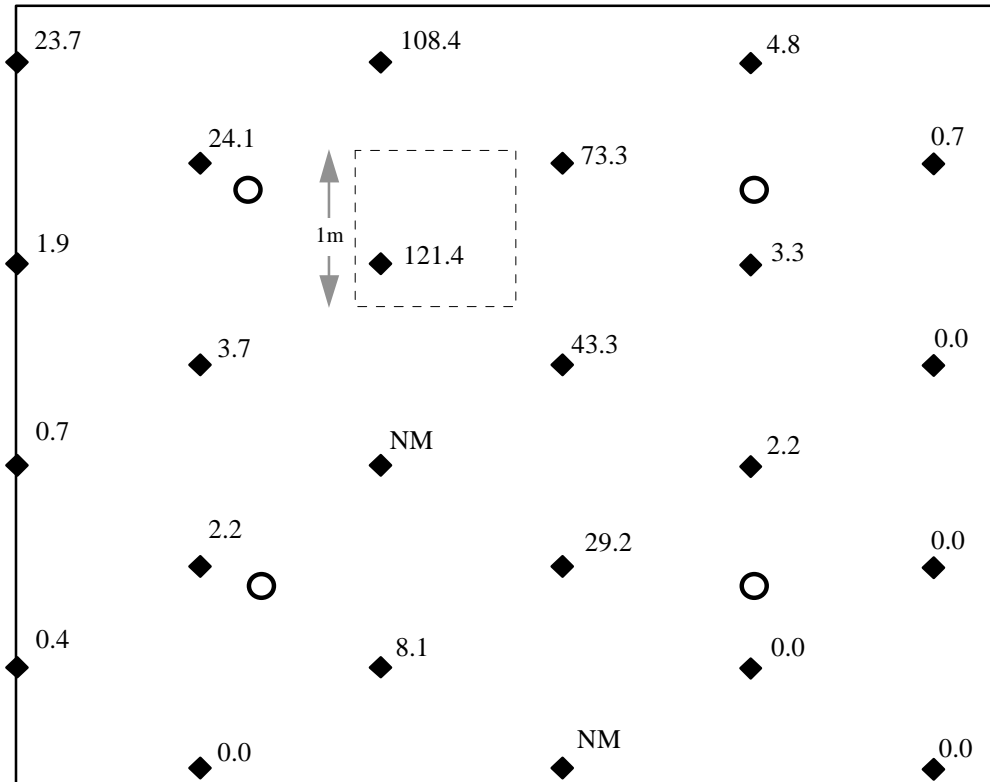


Figure 6. Activity distribution (Bq m⁻²) for largest potential area of elevated activity (test case 1).



- Location of potential area of elevated activity
- Location of uncollimated measurement
- ◆ Location of collimated measurement and measured surface activity (kBq m⁻²) (NM indicates that no measurement was made at that location)

Figure 7. Analysis of *in situ* measurements in a 5 m x 6 m room with ¹³⁷Cs contamination on the floor (test case 2). Data from collimated measurements, location of uncollimated measurements, and location of potential area of elevated activity identified by the code.

A PPENDIX A

THE PRINCIPLE OF MAXIMUM ENTROPY

The entropy of the distribution $f(x)$, in the form given in Skilling (1989), is given by

$$S = - \int \{ f(x) \log (f(x)/f(x)^{\text{def}}) + f(x)^{\text{def}} - f(x) \} dx, \quad (\text{A.1})$$

where $f(x)^{\text{Def}}$ is the default distribution. This is a generalization of the usual cross-entropy formula

$$S_{\text{CE}} = - \int \{ f(x) \log (f(x)/f(x)^{\text{def}}) \} dx, \quad (\text{A.2})$$

which appears as the first term in (A.1). The second term in (A.1) is an additive constant that ensures that the global maximum of S (at $f(x) = f(x)^{\text{def}}$) is zero, a sometimes convenient but not essential requirement. The third term in (A.1) ensures that in the absence of any other constraints S is maximized at $f(x) = f(x)^{\text{def}}$ (rather than just being proportional).

Before we go on, a note on terminology: some authors use the term entropy when referring to what we call here cross-entropy, while others use the term cross-entropy for the negative of S_{CE} and as a result end up with the principle of *minimum* cross-entropy. Other names for S_{CE} (or its negative) are relative entropy, direct divergence, expected weight of evidence, Kullback-Leibler distance, and Kullback number. To add to the confusion (and to keep the arguments as close to the standard literature as possible), when discussing the maximum entropy principle in this section, we will be referring to maximization of S_{CE} , not of S . For an axiomatic derivation of S (rather than S_{CE}), we refer the reader to Skilling (1989).

Historically, the concept of entropy originated in thermodynamics, but its generalization plays an important role in information theory, probability theory, and statistical inference. The maximum entropy principle provides a rule for determining a positive, additive distribution given definite but incomplete constraints. The probability distribution $\text{pr}(x)$ of a variable x is an example of a positive, additive distribution. It is positive by construction, and it is additive in the sense

that the overall probability in a domain equals the sum of the probabilities in any decomposition into subdomains. In our case, the positive, additive distribution to be determined is the distribution of activity $f(x)$, and the constraints are the measurements and the experimental errors associated with them. The maximum entropy principle states that one should choose from all of the distributions $f(x)$ that satisfy the set of constraints the one for which S_{CE} is maximal. A parallel procedure is used in statistical mechanics, where given constraints on the total energy and number of particles of a gas, entropy maximization is used to infer a distribution that is uniform in space and Maxwellian in velocity. There are several approaches that lead to the maximum entropy principle, and we summarize two of them below.

In the original derivation by Jaynes (1957, 1968), the use of the maximum entropy principle is justified on the basis of the cross-entropy's unique properties as an uncertainty measure. Arguments originating in information theory show that the Shannon entropy,

$$S_{SH} = - \sum f_i \log (f_i / f_i^{def}) \quad (A.3)$$

is the appropriate measure of the *amount of uncertainty expressed in a distribution*. These arguments follow from requiring three consistency conditions that are generally accepted as requirements for an uncertainty measure. Expression (A.3) for the case $f_i^{def} = 1$ was originally derived by Shannon (1948) for discrete distributions. Jaynes (1968) showed that (A.2) can be derived by generalizing to continuous distributions, in which case $f(x)^{def}$ appears as an “invariant measure” function (as well as playing the role of default distribution). Once a measure of the amount of uncertainty is established, it then seems reasonable to choose from all distributions $f(x)$ satisfying a set of constraints the one for which S_{CE} is maximal, since this is the distribution that “assumes the least” about the unknown $f(x)$. While agreeing with what is known, it will be maximally noncommittal with regard to all information except the data, and, therefore, the most unbiased estimate.

An independent justification that makes no reference to information theory is given by Shore and Johnson. Jaynes (1957) had already remarked that inferences made using any other information measure than the entropy may lead to contradictions. Shore and Johnson (1980) considered the consequences of requiring that methods of inference be self-consistent. They introduced four axioms that were all based on one fundamental principle: if a problem can be

solved in more than one way, the results should be consistent. Informally, they stated their axioms as follows:

- I. *Uniqueness*: The result should be unique,
- II *Invariance*: The choice of coordinate system should not matter,
- III. *System independence*: It should not matter whether one account for independent information about independent systems separately in terms of different densities or together in terms of a joint density, and
- IV. *Subset independence*: It should not matter whether one treats an independent subset of system states in terms of a separate different conditional density or in terms of the full system density.

Shore and Johnson go on to show that given information in the form of a set of constraints on expected values, there is only one distribution satisfying the set of constraints that can be chosen by a procedure that satisfies the four axioms, and this unique distribution can be obtained by maximizing S_{CE} . Therefore, if a method of inference is based on a variational principle, maximizing any function but the cross-entropy will lead to inconsistencies unless that function and entropy have identical maxima (any monotonic function of the entropy will work, for example).

A PPENDIX B

LISTING OF THE PROGRAM ISD97

PROGRAM ISD97

C
C
C
C
C

A computer program that analyzes data from a series of *in situ* measurements on a grid and identifies potential localized areas of elevated activity.

```
INTEGER M, NX, NY, NXY, MNXY, ISP
INTEGER DETNUM, PE
INTEGER MNUM(1000)
REAL A, HSA
REAL D(1000), S(1000), X(1000), Y(1000)
CHARACTER DH*8, EH*8
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY
```

C
C

```
PRINT*, ' NAME OF FILE WITH INPUT DATA? (USE 8 CHARACTERS) : '
READ*, DH
PRINT*, ' NAME OF OUTPUT FILE? (USE 8 CHARACTERS) : '
READ*, EH
PRINT*, ' AREA OF ELEVATED AREA? (IN SQUARE METERS, e. g. : 1.0) : '
READ*, HSA
```

C
C
C
C
C
C
C
C
C
C
C

Next the following data is read in from the file DH:

- (1) M is the number of measurements.
- (2) The ground is made up of (NX, NY) cells.
- (3) A is the size of each cell (in cm).
- (4) ISP is used by several subroutines. It is set up normally equal to one half the number of cells between measurements (or slightly more if the number of cells between measurements is not even). Must have $ISP > 0$. ISP is used in the subroutine PEAKXELS to fix the radius of the circle on which the integration over counts is done.

C (5) DETNUM is the detector number.
 C (6) PE is an integer, the photon energy.
 C (7) MNUM is the measurement number. Must Have MNUM(I) > 0.
 C (8) X, Y are the coordinates of measurements (in cm).
 C (9) D are the measurements (in cpm).
 C (10) S are the corresponding sigmas for each measurement.

C

OPEN (11, FILE=DH, STATUS=' OLD')

C

READ (11, FMT=*) M, NX, NY

READ (11, FMT=*) A

READ (11, FMT=*) ISP

READ (11, FMT=*) DETNUM

READ (11, FMT=*) PE

DO 1 I=1, M

 READ (11, FMT=*) MNUM(I), X(I), Y(I), D(I), S(I)

1

CONTINUE

CLOSE (11)

C

MNXY = M*NX*NY

NXY = NX*NY

C

CALL ISD(DH, EH, HSA, X, Y, D, S, DETNUM, PE, MNUM)

C

END

C

C

C

SUBROUTINE ISD(DH, EH, HSA, X, Y, D, S, DETNUM, PE, MNUM)

C

INTEGER I, II, K, L, IG

INTEGER IHP, KL, KU, LL, LU

INTEGER M, NX, NY, NXY, MNXY, ISP, HSS, G1, G2

INTEGER DETNUM, PE, NSD, NUM

INTEGER MNUM(1000)

REAL C, T, Z, ZAVG, MM, C1, CZ, CME

REAL HHP, PP, CSQ

REAL A, HHPNN, HSA

REAL DAVG, DBKG, SB, XMAX, XMIN, YMAX, YMIN, PPMCF, EAM, SUM1, SUM2

```

REAL D(M), DSORT(M), EZ(M), E(M), S(M), F(NXY)
REAL ERR(M), FP(NXY), PFE(M), RKHS(M), RLHS(M)
REAL MHS(M), CHISQ(M)
REAL B(MNXY), X(M), Y(M), ZKL(NXY)
REAL ZKLP(NXY)
REAL FPF(NXY), EF(M)
REAL FMAP(NXY)
REAL LAMBDA(M), LME(M), BI(M)
CHARACTER DH*8, EH*8, CH*12, TH*12, WH*12
CHARACTER UNITS*10
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY

```

C
C
C
C
C

Elevated Areas are modeled as squares of HSS cells per side.
HSS is an integer. Calculate how many cells per elevated area.
Calculate new value of HSA from the number of cells per elevated area.

```

HSS = FLOOR((SQRT(HSA))/(0.01*A))
HSA = (0.01*A*HSS)**2
PRINT*, ' AREA OF ELEVATED AREA (IN SQUARE METERS) SET TO : ', HSA
PRINT*, ' '

```

C
C
C
C

```
MM = FLOAT(M)
```

The user has a choice of two source distributions.

```

PRINT*, ' SOURCE DISTRIBUTION? (UNIFORM=1, PLANE=2) : '
READ*, NSD

```

C
C
C

Calculate the response matrix B.

```
CALL RESPONSE(B, A, X, Y, DETNUM, PE, UNITS, NSD, PPMCF)
```

C
C
C
C
C

Set the background activity distribution ZKL to a user chosen value Z (choices 3 or 4 are recommended). T is the total number of counts.

```

T = 0.0
DO 105 I=1, M

```

```

      T = T + D(I)
105  CONTINUE
      DAVG = T/MM
C
      PRINT*, ' AVERAGE COUNT RATE = ', DAVG
      PRINT*, ' '
C
      SB = 0.0
      DO 107 I=1, MNXY
          SB = SB + B(I)
107  CONTINUE
C
      ZAVG = T/SB
      Z = ZAVG
      DBKG = DAVG
C
      PRINT*, ' AVERAGE ACTIVITY (', UNITS, ') = ', ZAVG
      PRINT*, ' '
C
108  PRINT*, ' COUNT RATE TO USE TO SET A BACKGROUND ACTIVITY? : '
      PRINT*, ' 0 - THE AVERAGE VALUE '
      PRINT*, ' 1 - THE MEDIAN VALUE '
      PRINT*, ' 2 - THE COUNT RATE FROM THE LOWEST MEASUREMENT '
      PRINT*, ' 3 - THE WEIGHTED AVERAGE THAT MINIMIZES THE CHI SQUARE '
      PRINT*, ' 4 - A USER DETERMINED COUNT RATE : '
      READ*, IG
      IF (IG .EQ. 1) THEN
          DO 109 I=1, M
              DSORT(I) = D(I)
109  CONTINUE
              CALL HPSORT(M, DSORT)
              K = FLOOR(MM/2.0)
              DBKG = DSORT(K)
              Z = DBKG*(ZAVG/DAVG)
      ELSE IF (IG .EQ. 2) THEN
          DO 110 I=1, M
              DSORT(I) = D(I)
110  CONTINUE
              CALL HPSORT(M, DSORT)

```

```

    DBKG = DSORT(1)
    Z = DBKG*(ZAVG/DAVG)
ELSE IF (IG .EQ. 3) THEN
    DO 116 I=1, M
        BI(I) = 0.0
        DO 114 K=1, NX
            DO 112 L=1, NY
                BI(I) = BI(I) + B(NXY*(I-1)+NY*(K-1)+L)
112          CONTINUE
114          CONTINUE
116          CONTINUE
            SUM1 = 0.0
            SUM2 = 0.0
            DO 118 I=1, M
                SUM1 = SUM1 + D(I)*BI(I)/(S(I)**2)
                SUM2 = SUM2 + (BI(I)/S(I))**2
118          CONTINUE
            Z = SUM1/SUM2
            DBKG = Z*SB/MM
ELSE IF (IG .EQ. 4) THEN
    PRINT*, ' ENTER COUNT RATE (>0) IN cpm : '
    READ*, DBKG
    IF (DBKG .LE. 0.0) THEN
        PRINT*, ' COUNT RATE ENTERED MUST BE GREATER THAN ZERO '
        PRINT*, ' '
        GO TO 108
    END IF
    Z = DBKG*(ZAVG/DAVG)
END IF
C
PRINT*, ' BACKGROUND ACTIVITY (', UNITS, ') = ', Z
PRINT*, ' '
C
DO 211 K=1, NXY
    F(K) = Z
    ZKL(K) = Z
211 CONTINUE
C
C Calculate the CZ, the chi square of the background distribution.

```

```

C
  C1 = 0.0
  DO 216 I=1, M
    EZ(I) = 0.0
    DO 214 K=1, NX
      DO 212 L=1, NY
        EZ(I) = EZ(I) + B(NXY*(I-1)+NY*(K-1)+L)*F(NY*(K-1)+L)
212      CONTINUE
214      CONTINUE
        C1 = C1+((D(I)-EZ(I))**2)/(S(I)**2)
216      CONTINUE
C
  CZ = C1
C
  PRINT*, ' CHI SQUARE/INITIAL GUESS    = ', C1
  PRINT*, ' '
C
C  Initialize the lambdas to zero
C
  DO 218 I=1, M
    LAMBDA(I) = 0.0
218  CONTINUE
C
  NUM = 0
  CALL MAXENT(LAMBDA, F, ZKL, D, E, S, B, CME, NUM)
C
C  Save the lambdas from the MaxEnt deconvolution.
C
  DO 220 I=1, M
    LME(I) = LAMBDA(I)
220  CONTINUE
C
C  The FMAP is equal to the background level.
C  FMAP will be used to construct a map with a composite of elevated areas
C  placed on top of a map with background activity Z.
C
  DO 224 K=1, NXY
    FMAP(K) = Z
224  CONTINUE

```



```

C
C   The calculation of the locations and strengths of potential elevated
C   areas starts here.  The subroutine PEAKXELS integrates the total
C   excess activity (counts - bkg) surrounding each measurement point and
C   places the result in the array PFE.
C
CALL PEAKXELS(Z, F, PFE, X, Y)
C
C   For all PFE>0, the optimal position and magnitude for the
C   potential elevated area close to that PFE are determined.
C   FINHS calculates the position, MAGPS or MAGPSNN the magnitude.
C   PP is used to keep track of the intensity of the largest elevated area.
C
PP = 0.0
C
DO 305 I=1, M
  MHS(I) = 0.0
  RKHS(I) = 0.0
  RLHS(I) = 0.0
  CHISQ(I) = 0.0
  IF (PFE(I) .GT. 0.0) THEN
    PRINT*, ' '
    PRINT*, ' CALCULATING PARAMETERS NEAR MEASUREMENT' , MNUM(I)
    PRINT*, ' '
    DO 304 K=1, NXY
      FP(K) = ZKL(K)
      ZKLP(K) = ZKL(K)
304 CONTINUE
C
CALL FINDHS(X, Y, I, HSS, PFE, RKP, RLP, Z)
RKHS(I) = RKP
RLHS(I) = RLP
CALL MAGPS(D, S, RKP, RLP, I, HHP, CSQ, B, X, Y, ZKL, HSS)
CALL MAGPSNN(D, S, RKP, RLP, HHPNN, C1, B, A, X, Y, ZKL, HSS)
IF (HHPNN .GT. HHP) THEN
  HHP = HHPNN
  CSQ = C1
END IF
IF (HHP .LT. 1.0) THEN

```

```

      HHP = 1.0
    END IF

C
C   The elevated area is added to the FP and to the ZKLP (which were
C   set equal to the background activity).  These will be used in a final
C   MaxEnt deconvolution with background + elevated area.
C

      CALL MAKEHS(RKP, RLP, FP, HHP, X, Y, HSS)
      CALL MAKEHS(RKP, RLP, ZKLP, HHP, X, Y, HSS)

C
C   A MaxEnt deconvolution of the FP using as default distribution the ZKLP.
C
C   Initialize the lambdas - set them equal to the LME.
C

      DO 318 III=1, M
        LAMBDA(III) = LME(III)
318    CONTINUE

C

      NUM = 1
      CALL MAXENT(LAMBDA, FP, ZKLP, D, E, S, B, C, NUM)

C
C   The final magnitude of the elevated area is calculated.
C

      G1 = FLOOR(0.5*HSS)
      G2 = CEILING(0.5*HSS)
      RK = NINT(RKP)
      RL = NINT(RLP)
      KL = RK - G1
      KU = RK + G1
      LL = RL - G1
      LU = RL + G1
      IF (G1 .EQ. G2) THEN
        RK = FLOOR(RKP)
        RL = FLOOR(RLP)
        KL = RK - (G1 - 1)
        KU = RK + G1
        LL = RL - (G1 - 1)
        LU = RL + G1
      END IF

```

C

```
HHP = 0.0  
DO 332 K=KL, KU  
  DO 330 L=LL, LU  
    HHP = HHP + FP(NY*(K-1)+L)/(Z*HSS*HSS)
```

330

```
CONTINUE
```

332

```
CONTINUE
```

C

```
CSQ = C  
MHS(I) = HHP  
CHISQ(I) = CSQ
```

C

C

Print to screen the parameters of this potential ELEVATED AREA.

C

```
PRINT*, ' '  
PRINT*, ' ELEVATED AREA (?) /MEAS. NUMBER = ', MNUM(I)  
PRINT*, ' ELEVATED AREA (?) /X COORDINATE = ', RKP  
PRINT*, ' ELEVATED AREA (?) /Y COORDINATE = ', RLP  
PRINT*, ' ELEVATED AREA (?) /SIZE (x BKG) = ', HHP  
PRINT*, ' ELEVATED AREA (?) /CHI SQUARE = ', CSQ  
PRINT*, ' '
```

C

```
CALL MAKEHSMAP(RKP, RLP, FMAP, HHP, X, Y, HSS, Z)
```

C

C

If this elevated area is the largest one so far, the FP are saved as FPF, and IHP is set to I. The E are saved as as EF.

C

C

```
IF (MHS(I) .GT. PP) THEN  
  IHP = I  
  DO 340 II=1, M  
    EF(II) = E(II)  
340 CONTINUE  
  DO 344 K=1, NXY  
    FPF(K) = FP(K)  
344 CONTINUE  
  PP = MHS(I)  
END IF
```

C

```
END IF
```

```

305 CONTINUE
C
C Print to screen the parameters of the largest potential elevated
C area found.
C
RKP = RKHS(IHP)
RLP = RLHS(IHP)
HHP = MHS(IHP)
C1 = CHISQ(IHP)
C
PRINT*, ' '
PRINT*, ' PARAMETERS OF LARGEST POTENTIAL ELEVATED AREA FOUND: '
PRINT*, ' '
PRINT*, ' MEAS. NUMBER = ', MNUM(IHP)
PRINT*, ' X COORDINATE = ', RKP
PRINT*, ' Y COORDINATE = ', RLP
PRINT*, ' SIZE (x BKG) = ', HHP
PRINT*, ' CHI SQUARE = ', C1
PRINT*, ' '
C
C Set the FP equal to the FPF.
C
DO 432 K=1, NXY
    FP(K) = FPF(K)
432 CONTINUE
C
C *** OUTPUT ***
C
DO 905 I=1, M
    ERR(I) = (EF(I) - D(I)) / S(I)
905 CONTINUE
C
CH = EH//'.MXE'
TH = EH//'.TBL'
WH = EH//'.ALL'
C
OPEN (10, FILE=CH, STATUS='NEW')
OPEN (12, FILE=EH, STATUS='NEW')

```

```

OPEN (14, FILE=TH, STATUS=' NEW )
OPEN (16, FILE=WH, STATUS=' NEW )
C
C Set the limits of the activity maps to the rectangle bounded by the
C maxima and minima of X, Y (in units of cells) plus or
C minus ISP.
C
XMAX = X(1)
XMIN = X(1)
YMAX = Y(1)
YMIN = Y(1)
C
DO 910 I=1, M
  IF (X(I) .GT. XMAX) THEN
    XMAX = X(I)
  END IF
  IF (X(I) .LT. XMIN) THEN
    XMIN = X(I)
  END IF
  IF (Y(I) .GT. YMAX) THEN
    YMAX = Y(I)
  END IF
  IF (Y(I) .LT. YMIN) THEN
    YMIN = Y(I)
  END IF
910 CONTINUE
C
KU = CEILING(XMAX/A) + ISP
KL = FLOOR(XMIN/A) - ISP
LU = CEILING(YMAX/A) + ISP
LL = FLOOR(YMIN/A) - ISP
C
IF (KU .GT. NX) THEN
  KU = NX
END IF
IF (KL .LT. 1) THEN
  KL = 1
END IF
IF (LU .GT. NY) THEN

```

```

    LU = NY
END IF
IF (LL .LT. 1) THEN
    LL = 1
END IF
C
DO 930 K=KL, KU
    WRITE (10, 920) (F(NY*(K-1)+L), L = LL, LU)
    WRITE (12, 920) (FP(NY*(K-1)+L), L = LL, LU)
    WRITE (16, 920) (FMAP(NY*(K-1)+L), L = LL, LU)
920   FORMAT( 1000 (2X, E8. 2) )
930   CONTINUE
C
    WRITE (14, 932) EH
932   FORMAT(/2X, 'ACTIVITY ARRAY IS STORED IN FILE : ', A)
C
    WRITE (14, 933) DETNUM
933   FORMAT(/2X, 'DETECTOR NUMBER : ', I10)
    WRITE (14, 934) PE
934   FORMAT(2X, 'PHOTON ENERGY (KeV) = ', I5)
C
    WRITE (14, 935) DAVG
935   FORMAT(/2X, 'AVERAGE COUNT RATE (cpm) = ', 1P, E10. 4)
    WRITE (14, 936) UNITS, ZAVG
936   FORMAT(2X, 'AVERAGE ACTIVITY (' , A10, ') = ', 1P, E10. 4)
C
    WRITE (14, 944) DBKG
944   FORMAT(/2X, 'AVERAGE BACKGROUND COUNT RATE (cpm) SET TO : ', 1P,
1E10. 4)
    WRITE (14, 945) UNITS, Z
945   FORMAT(2X, 'BACKGROUND ACTIVITY (' , A10, ') SET TO : ', 1P, E10. 4)
    IF (PPMCF .GT. 0. 0) THEN
        WRITE (14, 947) PPMCF
947   FORMAT(2X, 'CONVERSION FACTOR (pCi/g TO ppm) USED = ', 1P, E10. 4,
1' ppm/(pCi/g)')
    END IF
C
    WRITE (14, 948) CZ
948   FORMAT(/2X, 'CHI SQUARE / BACKGROUND ACTIVITY = ', 1P, E10. 4)

```

```

WRITE (14, 949) CME
949  FORMAT(2X, ' CHI SQUARE / MAXIMUM ENTROPY SOLUTION = ', F10. 4)
C
WRITE (14, 950) IHP
950  FORMAT(/2X, ' LARGEST POTENTIAL ELEVATED AREA NEAR MEASUREMENT : '
1, I3)
WRITE (14, 951) HSA
951  FORMAT(2X, ' AREA (m^2) = ', F10. 4)
WRITE (14, 952) C1
952  FORMAT(2X, ' CHI SQUARE = ', F10. 4)
WRITE (14, 954) HHP
954  FORMAT(2X, ' MAGNITUDE (x BACKGROUND) = ', F10. 4)
EAM = HHP*Z
WRITE (14, 955) UNITS, EAM
955  FORMAT(2X, ' MAGNITUDE (', A10, ') = ', 1P, E10. 4)
C
WRITE (14, 956)
956  FORMAT(/2X, ' NUM/MEAS  CTS/MEAS  CTS/CALC      (E-D)/S=' )
DO 960 I=1, M
    WRITE (14, 958) MNUM(I), D(I), EF(I), ERR(I)
958    FORMAT(2X, I8, 2X, 1P, E8. 2, 2X, E8. 2, 2X, 0P, F8. 3)
960  CONTINUE
C
WRITE (14, 970)
970  FORMAT(/2X, ' NUM/MEAS      X/HSPT      Y/HSPT      MAG HSPT      CHI SQR' )
C
DO 985 I=1, M
    WRITE (14, 980) I, RKHS(I), RLHS(I), MHS(I), CHISQ(I)
980    FORMAT(2X, I8, 2X, F8. 2, 2X, F8. 2, 2X, F8. 1, 2X, F8. 2)
985  CONTINUE
C
CLOSE (10, STATUS=' KEEP' )
CLOSE (12, STATUS=' KEEP' )
CLOSE (14, STATUS=' KEEP' )
CLOSE (16, STATUS=' KEEP' )
C
PRINT*, ' '
PRINT*, ' Activity Array is Stored in File : ', EH
C

```

9999 RETURN
END

C
C
C

SUBROUTINE PEAKXELS(Z, F, PFE, X, Y)

C

INTEGER M, NX, NY, NXY, I, K, L, KL, KU, LL, LU, MHD, MHDSQ, RR
INTEGER ISP
REAL XX, YY, Z
REAL A
REAL F(NXY), PFE(M)
REAL X(M), Y(M)
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY

C

DO 20 I=1, M
MHD = ISP
MHDSQ = MHD**2
XX = X(I)/A
YY = Y(I)/A
KL = INT(XX) - MHD
KU = INT(XX) + MHD
LL = INT(YY) - MHD
LU = INT(YY) + MHD

C

IF (KL .LT. 1) THEN
KL = 1
END IF
IF (KU .GT. NX) THEN
KU = NX
END IF
IF (LL .LT. 1) THEN
LL = 1
END IF
IF (LU .GT. NY) THEN
LU = NY
END IF

C


```

      PFE(I) = 0.0
      DO 10 K=KL, KU
        DO 8 L=LL, LU
          RR = ((INT(XX) - K)**2) + ((INT(YY) - L)**2)
          IF (RR .LE. MHDSQ) THEN
            PFE(I) = PFE(I) + F(NY*(K-1)+L) - Z
          END IF
8          CONTINUE
10         CONTINUE
20        CONTINUE
C
      RETURN
      END
C
C *****
C
SUBROUTINE FINDHS(X, Y, IHP, HSS, PFE, RKP, RLP, Z)
C
C FINDHS calculates the optimal coordinates (RKP, RLP) of the elevated
C area.
C
      INTEGER M, NX, NY, NXY, MNXY, ISP, IHP, HSS, IHPK, IHPL, K
      INTEGER I, J, KK, LL, KKL, KKL, KKL, LLL, LLU
      REAL DSQ, R2, RR, KO, XH, YH
      REAL A, EPSSQMIN, RKP, RLP, Z
      REAL X(M), XX(M), Y(M), YY(M), PFE(M), PPFE(M)
      REAL RRIJ(M, M), RRHI(M), PHS(NX, NY), EPS(M), EPSSQ(NX, NY)
      COMMON/AISP/A, ISP
      COMMON/MN/M, NX, NY, MNXY, NXY
C
C R2 is defined in such a way that for a square grid it includes
C the eight closest measurement points (DSQ is the square of the
C distance from the center of the square to any of the corners;
C R2 is the square of {1.5*the distance to closest measurement}).
C (For a triangular grid it includes the six closest measurements).
C
      DSQ = 2.0*(2.0*A*FLOAT(ISP))**2
      R2 = 1.125*DSQ
C

```

```

C      Calculate (XX,YY), the x and y coordinates for measurement points
C      with PFE>0. Evaluate K, the number of such measurement points.
C
      K = 0
      DO 10 I=1, M
        RR = (X(IHP)-X(I))**2 + (Y(IHP)-Y(I))**2
        IF (RR .LE. R2) THEN
          IF (PFE(I) .GE. 0.0) THEN
            K = K+1
            XX(K) = X(I)
            YY(K) = Y(I)
            PPF(K) = PFE(I)
          END IF
        END IF
      CONTINUE
      KO = FLOAT(K)
C
C      Calculate the square of the distance between detectors that are 100cm
C      above the ground and points on the ground directly under the
C      detectors. Call it RRIJ(I, J).
C
      DO 14 I=1, K
        DO 12 J=1, K
          RRIJ(I, J) = (XX(I)-XX(J))**2 + (YY(I)-YY(J))**2 + 10000.0
        CONTINUE
      CONTINUE
C
      IHPK = NINT(X(IHP)/A)
      IHPL = NINT(Y(IHP)/A)
      KKL = IHPK - 3*ISP
      KKU = IHPK + 3*ISP
      LLL = IHPL - 3*ISP
      LLU = IHPL + 3*ISP
C
      IF (KKL .LT. 1) THEN
        KKL = 1
      END IF
      IF (KKU .GT. NX) THEN
        KKU = NX

```

```

END IF
IF (LLL .LT. 1) THEN
  LLL = 1
END IF
IF (LLU .GT. NY) THEN
  LLU = NY
END IF
C
DO 28 KK=KKL, KKU
  DO 26 LL=LLL, LLU
    XH = A*(FLOAT(KK))
    YH = A*(FLOAT(LL))
    RR = (X(IHP) - XH)**2 + (Y(IHP) - YH)**2
    IF (RR .LE. R2) THEN
C
C      Calculate the optimal activity PHS for a elevated area (point source)
C      at cell coordinates (KK, LL).
C
      PHS(KK, LL) = 0.0
      DO 18 I=1, K
        RRHI(I) = (XH - XX(I))**2 + (YH - YY(I))**2 + 10000.0
        DO 16 J=1, K
          PHS(KK, LL) = PHS(KK, LL) + (RRHI(I) * PPFE(J) / RRIJ(I, J)) / KO
16        CONTINUE
18      CONTINUE
C
C      Calculate the additional activity EPS needed at each measurement
C      point to keep the number of counts invariant in the
C      presence of the elevated area.
C
      DO 22 I=1, K
        EPS(I) = 0.0
        DO 20 J=1, K
          EPS(I) = EPS(I) + RRHI(I) * PPFE(I) / RRIJ(I, J)
20        CONTINUE
          EPS(I) = EPS(I) + PHS(KK, LL)
22      CONTINUE
C
C      Calculate EPSSQ(KK, LL), the sum of the square of the EPSs.

```

```

C
      EPSSQ(KK, LL) = 0.0
      DO 24 I=1, K
          EPSSQ(KK, LL) = EPSSQ(KK, LL) + (EPS(I)) **2
24      CONTINUE
C
      END IF
26      CONTINUE
28      CONTINUE
C
C      Place the elevated area at the cell coordinates (KK, LL) that
C      minimize EPSSQ(KK, LL). Set (RKP, RLP)=(XH/A, YH/A).
C
      EPSSQMIN = EPSSQ(IHPK, IHPL)
      DO 38 KK=KKL, KKL
          DO 36 LL=LLL, LLU
              XH = A*FLOAT(KK)
              YH = A*FLOAT(LL)
              RR = (X(IHP) - XH) **2 + (Y(IHP) - YH) **2
              IF (RR .LE. R2) THEN
                  IF (EPSSQ(KK, LL) .LE. EPSSQMIN) THEN
                      EPSSQMIN = EPSSQ(KK, LL)
                      RKP = XH/A
                      RLP = YH/A
                  END IF
              END IF
          END IF
36      CONTINUE
38      CONTINUE
C
      RETURN
      END
C
C      *****
C
      SUBROUTINE MAGPS(D, S, RKP, RLP, IHP, HHP, CSQ, B, X, Y, ZKL, HSS)
C
      INTEGER M, NX, NY, NXY, MNXY, I, K, L, KP, LP, IHP
      INTEGER ISP, HSS
      REAL DSQ, RR, R2, RKP, RLP, SUM1, SUM2, HHP, HPS, CSQ

```

```

REAL A
REAL D(M) , S(M) , E(M) , DE(M) , FD(NXY)
REAL B(MNXY) , X(M) , Y(M)
REAL ZKL(NXY)
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY

C
C   MAGPS scales the point source at (RKP, RLP) to the optimal height,
C   which is given by the multiplicative factor HPS.
C   HPS is calculated by minimizing the chi square FOR THE
C   MEASUREMENTS NEAR THE ELEVATED AREA under the assumption that the
C   surface activity is given by the ZKL with a point source
C   added at (RKP, RLP).
C
C   Set the FD = ZKL
C
DO 110 K=1, NXY
    FD(K) = ZKL(K)
110 CONTINUE
C
C   Calculate the E due to the background.
C
DO 6 I=1, M
    E(I) = 0.0
    DO 4 K=1, NX
        DO 2 L=1, NY
            E(I) = E(I) + B(NXY*(I-1)+NY*(K-1)+L) *FD(NY*(K-1)+L)
2        CONTINUE
4        CONTINUE
6        CONTINUE
C
C   Calculate the DE due to activity at the point source.
C
KP = INT(RKP)
LP = INT(RLP)
C
DO 10 I=1, M
    DE(I) = B(NXY*(I-1)+NY*(KP-1)+LP) *FD(NY*(KP-1)+LP)
10 CONTINUE

```

```

C
C   Calculate the optimal activity of the point source.
C
DSQ = 2.0*(2.0*A*FLOAT(ISP))**2
R2 = 1.125*DSQ
SUM1 = 0.0
SUM2 = 0.0
C
DO 20 I=1, M
  RR = (X(IHP)-X(I))**2 + (Y(IHP)-Y(I))**2
  IF (RR .LE. R2) THEN
    SUM1 = SUM1 + DE(I)*(D(I)-E(I))/(S(I)**2)
    SUM2 = SUM2 + DE(I)*DE(I)/(S(I)**2)
  END IF
20 CONTINUE
C
HPS = 1.0 + SUM1/SUM2
C
C   Add the elevated area to the FD.
C
FD(NY*(KP-1)+LP) = FD(NY*(KP-1)+LP) + (HHP-1.0)*FD(NY*(KP-1)+LP)
C
C   Calculate the chi square for this point source + background.
C
CSQ = 0.0
DO 38 I=1, M
  E(I) = 0.0
  DO 37 K=1, NX
    DO 36 L=1, NY
      E(I) = E(I)+B(NXY*(I-1)+NY*(K-1)+L)*FD(NY*(K-1)+L)
36 CONTINUE
37 CONTINUE
  CSQ = CSQ+((D(I)-E(I))**2)/(S(I)**2)
38 CONTINUE
C
C   Set HPP = HPS/(area ELEVATED AREA in cells).
C
HHP = HPS/(HSS**2)
C

```

```

RETURN
END
C
C *****
C
SUBROUTINE MAGPSNN(D, S, RKP, RLP, HHP, CSQ, B, A, X, Y, ZKL, HSS)
C
INTEGER M, NX, NY, NXY, MNXY, I, K, L, RK, RL, NHS, HSS
REAL RR, RKP, RLP, HHP, HPS, CSQ, A
REAL XX, YY, RRN, EBKG, DE
REAL D(M), S(M), E(M), FD(NXY)
REAL B(MNXY), X(M), Y(M)
REAL ZKL(NXY)
COMMON/MN/M, NX, NY, MNXY, NXY
C
C HHP is calculated by fitting the point source's magnitude to the
C total counts at the nearest's measurement point - under the
C assumption that the surface activity is given by the ZKL
C with a point source added at (RKP, RLP).
C
C Set the FD = ZKL
C
DO 10 K=1, NXY
    FD(K) = ZKL(K)
10 CONTINUE
C
C Find the measurement point that is nearest to the elevated area.
C
XX = X(1) / A
YY = Y(1) / A
NHS = 1
RRN = (XX - RKP) ** 2 + (YY - RLP) ** 2
DO 12 I=2, M
    XX = X(I) / A
    YY = Y(I) / A
    RR = (XX - RKP) ** 2 + (YY - RLP) ** 2
    IF (RR .LT. RRN) THEN
        NHS = I
        RRN = RR

```

```

      END IF
12  CONTINUE
C
C  Calculate the EBKG for the NHS due to the background.
C
      EBKG = 0.0
      DO 24 K=1, NX
        DO 22 L=1, NY
          EBKG = EBKG + B(NXY*(NHS-1)+NY*(K-1)+L)*FD(NY*(K-1)+L)
22      CONTINUE
24  CONTINUE
C
C  Calculate the DE due to activity at the point source location.
C
      RK = INT(RKP)
      RL = INT(RLP)
C
      DE = DE + B(NXY*(NHS-1)+NY*(RK-1)+RL)*FD(NY*(RK-1)+RL)
C
C  Calculate the (largest) optimal activity of the point source
C  assuming  $E(NHS) = EBKG + (HPS-1)*DE = D(NHS) + S(NHS)$ .
C
      HPS = 1.0 + (D(NHS)+S(NHS) - EBKG) / DE
C
C  Add the point source to the FD(K, L).
C
      FD(NY*(RK-1)+RL) = FD(NY*(RK-1)+RL) + (HHP-1.0)*FD(NY*(RK-1)+RL)
C
C  Calculate the chi square for this point source + background.
C
      CSQ = 0.0
      DO 38 I=1, M
        E(I) = 0.0
        DO 37 K=1, NX
          DO 36 L=1, NY
            E(I) = E(I) + B(NXY*(I-1)+NY*(K-1)+L)*FD(NY*(K-1)+L)
36      CONTINUE
37  CONTINUE
      CSQ = CSQ + ((D(I) - E(I)) ** 2) / (S(I) ** 2)

```



```

38 CONTINUE
C
C Calculate HHP = HPS/(HSS**2).
C
HHP = HPS/(HSS**2)
C
RETURN
END
C
C *****
C
SUBROUTINE MAKEHS(RKP, RLP, FP, HHP, X, Y, HSS)
C
INTEGER K, L, KL, KU, LL, LU, NX, NY, NXY
INTEGER ISP, M, HSS, G1, G2
REAL RKP, RLP, RK, RL, HHP
REAL A
REAL FP(NXY)
REAL X(M), Y(M)
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY
C
C Model the elevated area with a (HSS**2)-cell area. MAKEHS scales
C the elevated area to the optimal height, which is given by the
C multiplicative factor HHP, and calculates the final FP.
C
C HHP was calculated in MAGPS or MAGPSNN.
C
G1 = FLOOR(0.5*HSS)
G2 = CEILING(0.5*HSS)
RK = NINT(RKP)
RL = NINT(RLP)
KL = RK - G1
KU = RK + G1
LL = RL - G1
LU = RL + G1
IF (G1 .EQ. G2) THEN
    RK = FLOOR(RKP)
    RL = FLOOR(RLP)

```

```

      KL = RK - (G1- 1)
      KU = RK + G1
      LL = RL - (G1- 1)
      LU = RL + G1
END IF
C
DO 34 K=KL, KU
  DO 32 L=LL, LU
    FP(NY*(K- 1)+L) = FP(NY*(K- 1)+L) + (HHP- 1. 0) *FP(NY*(K- 1)+L)
32  CONTINUE
34  CONTINUE
C
RETURN
END
C
C *****
C
SUBROUTINE MAKEHSMAP(RKP, RLP, FP, HHP, X, Y, HSS, Z)
C
INTEGER K, L, KL, KU, LL, LU, NX, NY, NXY
INTEGER ISP, M, HSS, G1, G2
REAL RKP, RLP, RK, RL, HHP
REAL A, DUMMY, Z
REAL FP(NXY)
REAL X(M), Y(M)
COMMON/AISP/A, ISP
COMMON/MN/M, NX, NY, MNXY, NXY
C
C Model the elevated area with a (HSS**2)- cell area. MAKEHS scales
C the elevated area to the optimal height, which is given by the
C multiplicative factor HHP, and calculates the final FP.
C
C HHP was calculated in MAGPS or MAGPSNN.
C
G1 = FLOOR(0. 5*HSS)
G2 = CEILING(0. 5*HSS)
RK = NINT(RKP)
RL = NINT(RLP)
KL = RK - G1

```

```

    KU = RK + G1
    LL = RL - G1
    LU = RL + G1
    IF (G1 .EQ. G2) THEN
        RK = FLOOR(RKP)
        RL = FLOOR(RLP)
        KL = RK - (G1- 1)
        KU = RK + G1
        LL = RL - (G1- 1)
        LU = RL + G1
    END IF
C
    DO 34 K=KL, KU
        DO 32 L=LL, LU
            DUMMY = FP(NY*(K- 1)+L)/Z
            IF (DUMMY .LT. HHP) THEN
                FP(NY*(K- 1)+L) = HHP*Z
            END IF
32        CONTINUE
34    CONTINUE
C
    RETURN
    END
C
C *****
C
SUBROUTINE hpsort(n, ra)
INTEGER n
REAL ra(n)
INTEGER i, ir, j, l
REAL rra
if (n.lt. 2) return
l=n/2+1
ir=n
10 continue
    if(l.gt. 1) then
        l=l- 1
        rra=ra(l)
    else

```

```

        rra=ra(ir)
        ra(ir)=ra(1)
        ir=ir-1
        if(ir.eq.1) then
            ra(1)=rra
            return
        endif
    endif
    i=l
    j=l+1
20    if(j.le.ir) then
        if(j.lt.ir) then
            if(ra(j).lt.ra(j+1))j=j+1
        endif
        if(rra.lt.ra(j)) then
            ra(i)=ra(j)
            i=j
            j=j+j
        else
            j=ir+1
        endif
        goto 20
    endif
    ra(i)=rra
    goto 10
END

```

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C

C

C

SUBROUTINE MAXENT(LAMBDA, F, FDEF, D, E, S, B, C, NUM)

C

INTEGER M, NX, NY, NXY, MNXY

INTEGER ITER, NUM

REAL C, FTOL

REAL LAMBDA(M), D(M), E(M), S(M)

REAL F(NXY), FDEF(NXY), B(MNXY), SUM(NX, NY)

COMMON/MN/M, NX, NY, MNXY, NXY

C

```

FTOL = 1.0E-20
C
IF (NUM .EQ. 0) THEN
  PRINT*, ' STARTING MAXIMUM ENTROPY DECONVOLUTION OF DATA '
  PRINT*, ' '
ELSE IF (NUM .GT. 0) THEN
  PRINT*, ' STARTING DECONVOLUTION FOR ACTIVITY DISTRIBUTION '
  PRINT*, ' OF BACKGROUND + ELEVATED AREA '
  PRINT*, ' '
END IF
C
IF (NUM .EQ. 0) THEN
  PRINT*, ' STARTING MINIMIZATION OF POTENTIAL FUNCTION '
  PRINT*, ' '
  CALL FRPRMN(LAMBDA, M, FTOL, ITER, FRET, FDEF, D, S, B)
  PRINT*, ' RESTARTING MINIMIZATION OF POTENTIAL FUNCTION '
  PRINT*, ' '
  CALL LMBDAS(FTOL, LAMBDA, ITER, FDEF, D, S, B)
ELSE IF (NUM .GT. 0) THEN
  CALL LMBDAS(FTOL, LAMBDA, ITER, FDEF, D, S, B)
END IF
C
DO 14 K=1, NX
  DO 12 L=1, NY
    SUM(K, L) = 0.0
    DO 10 I=1, M
      SUM(K, L) = SUM(K, L) + LAMBDA(I) * B(NXY*(I-1) + NY*(K-1) + L)
10    CONTINUE
12    CONTINUE
14  CONTINUE
C
DO 18 K=1, NX
  DO 16 L=1, NY
    F(NY*(K-1) + L) = FDEF(NY*(K-1) + L) * EXP(-SUM(K, L))
16  CONTINUE
18  CONTINUE
C
C = 0.0
DO 24 I=1, M

```

```

E(I) = 0.0
DO 22 K=1, NX
  DO 20 L=1, NY
    E(I) = E(I) + B(NXY*(I-1)+NY*(K-1)+L)*F(NY*(K-1)+L)
20  CONTINUE
22  CONTINUE
    C = C + ((D(I) - E(I)) **2) / (S(I) **2)
24  CONTINUE
C
IF (NUM .EQ. 0) THEN
  PRINT*, ' CHI SQUARE/MAXIMUM ENTROPY = ', C
  PRINT*, ' '
END IF
C
RETURN
END
C
C *****
C
FUNCTION FUNC(LAMBDA, FDEF, D, S, B)
C
INTEGER K, L, M, NX, NY, NXY
REAL OMEGA, SUM1, SUM2, SUM3, SUM4, SUM5
REAL LAMBDA(M), FDEF(NXY), D(M), S(M), B(MNXY)
COMMON/MN/M, NX, NY, MNXY, NXY
C
C FUNC is the potential function.
C
OMEGA = FLOAT(M)
C
SUM1 = 0.0
SUM5 = 0.0
DO 14 K=1, NX
  DO 12 L=1, NY
    SUM2 = 0.0
    DO 10 I=1, M
      SUM2 = SUM2 + LAMBDA(I)*B(NXY*(I-1)+NY*(K-1)+L)
10  CONTINUE
    SUM1 = SUM1 + FDEF(NY*(K-1)+L)*EXP(-SUM2)

```

```

        SUM5 = SUM5 + FDEF(NY*(K-1)+L)
12      CONTINUE
14      CONTINUE
C
      SUM3 = 0.0
      SUM4 = 0.0
      DO 16 I=1, M
        SUM3 = SUM3 + (S(I)*LAMBDA(I))**2
        SUM4 = SUM4 + D(I)*LAMBDA(I)
16     CONTINUE
C
      FUNC = -SUM1 - SQRT(OMEGA*SUM3) - SUM4 + SUM5
      FUNC = -FUNC
C
      RETURN
      END
C
C      *****
C
SUBROUTINE DFUNC(LAMBDA, GRADZ, FDEF, D, S, B)

INTEGER K, L, M, NX, NY, NXY, DUMMY
REAL OMEGA, A1, A2, SUM1, SUM2(NX, NY), SUM3
REAL LAMBDA(M), GRADZ(M), FDEF(NXY), D(M), S(M), B(MNXY)
COMMON/MN/M, NX, NY, MNXY, NXY

C
C      DFUNC is the gradient of the potential function.
C
      OMEGA = FLOAT(M)

C
      A1 = 0.0
      A2 = 0.0
      DO 8 I=1, M
        A1 = A1 + (S(I)*LAMBDA(I))**2
        A2 = A2 + (S(I))**2
8     CONTINUE

      DO 14 K=1, NX
        DO 12 L=1, NY

```

```

SUM2(K, L) = 0.0
DO 10 I=1, M
    SUM2(K, L) = SUM2(K, L) + LAMBDA(I) * B(NXY*(I-1) + NY*(K-1) + L)
10 CONTINUE
12 CONTINUE
14 CONTINUE
C

```

```

DO 30 I=1, M
    SUM1 = 0.0
    DO 18 K=1, NX
        DO 16 L=1, NY
            DUMMY = NXY*(I-1) + NY*(K-1) + L
            SUM1 = SUM1 + FDEF(NY*(K-1) + L) * EXP(-SUM2(K, L)) * B(DUMMY)
16 CONTINUE
18 CONTINUE
C

```

```

SUM3 = ((S(I)) ** 2) * (SQRT(OMEGA/A2))
IF (A1 .GT. 1.0E-20) THEN
    SUM3 = ((S(I)) ** 2) * LAMBDA(I) * (SQRT(OMEGA/A1))
END IF
C

```

```

GRADZ(I) = SUM1 - SUM3 - D(I)
GRADZ(I) = -GRADZ(I)
30 CONTINUE
C

```

```

RETURN
END
C

```

```

*****
C

```

```

SUBROUTINE LMBDAS(FTOL, LAMBDA, ITER, FDEF, D, S, B)

```

```

INTEGER K, L, M, NX, NY, NXY, ITER, ITERMAX
REAL OMEGA, LB(NX, NY), EPS
REAL TWOMU, Z, ZOLD, FTOL, P, C
REAL LAMBDA(M), LAMBDAOLD(M), DL, DLMIN
REAL FDEF(NXY), D(M), E(M), S(M), B(MNXY)
COMMON/MN/M, NX, NY, MNXY, NXY
C

```



```

OMEGA = FLOAT(M)
ITER = 1
ITERMAX = 250
EPS = 1.0E-10
DLMIN = 0.1
P = 0.01
DO 2 I=1, M
    LAMBDAOLD(I) = LAMBDA(I)
2 CONTINUE
ZOLD = FUNC(LAMBDA, FDEF, D, S, B)
C
DO 8 K=1, NX
    DO 6 L=1, NY
        LB(K, L) = 0.0
        DO 4 I=1, M
            LB(K, L) = LB(K, L) + LAMBDA(I) * B(NXY*(I-1) + NY*(K-1) + L)
4 CONTINUE
6 CONTINUE
8 CONTINUE
C = 0.0
DO 14 I=1, M
    E(I) = 0.0
    DO 12 K=1, NX
        DO 10 L=1, NY
            E(I) = E(I) + FDEF(NY*(K-1) + L) * EXP(-LB(K, L)) * B(NXY*(I-1) + NY*(K-1) + L)
10 CONTINUE
12 CONTINUE
    C = C + ((E(I) - D(I)) / S(I)) ** 2
14 CONTINUE
C
C Use the integral equation to modify the lambdas.
C
15 TWOMU = 0.0
DO 16 I=1, M
    TWOMU = TWOMU + (S(I) * LAMBDA(I)) ** 2
16 CONTINUE
TWOMU = SQRT(TWOMU / OMEGA)
C
DO 18 I=1, M

```

```

      LAMBDA(I) = (1.0 - P)*LAMBDA(I) + P*(E(I) - D(I))*TWOMU/(S(I)**2)
18  CONTINUE
      Z = FUNC(LAMBDA, FDEF, D, S, B)
C
      IF (Z .GT. ZOLD) THEN
          DO 24 I=1, M
              LAMBDA(I) = LAMBDAOLD(I)
24  CONTINUE
          IF (ABS(C - OMEGA) .LT. SQRT(OMEGA)) THEN
              RETURN
          END IF
          CALL FRPRMN(LAMBDA, M, FTOL, ITER, FRET, FDEF, D, S, B)
          RETURN
      END IF
C
      IF (ITER .GT. ITERMAX) THEN
          DO 26 I=1, M
              LAMBDA(I) = LAMBDAOLD(I)
26  CONTINUE
          IF (ABS(C - OMEGA) .LT. SQRT(OMEGA)) THEN
              RETURN
          END IF
          CALL FRPRMN(LAMBDA, M, FTOL, ITER, FRET, FDEF, D, S, B)
          RETURN
      END IF
C
      PRINT*, ' SUBROUTINE LMBDAS/NUMBER OF ITERATIONS : ', ITER
      PRINT*, ' '
C
      IF (2.0*ABS(Z - ZOLD) .LE. FTOL*(ABS(Z) + ABS(ZOLD) + EPS)) THEN
          RETURN
      END IF
C
      DO 34 K=1, NX
          DO 32 L=1, NY
              LB(K, L) = 0.0
              DO 30 I=1, M
                  LB(K, L) = LB(K, L) + LAMBDA(I)*B(NXY*(I - 1) + NY*(K - 1) + L)
30  CONTINUE

```



```

INTEGER iter, n, NMAX, ITMAX
REAL fret, ftol, p(n), EPS, func
EXTERNAL func
PARAMETER (NMAX=500, ITMAX=200, EPS=1. e- 10)
CU  USES dfunc, func, linmi n
    INTEGER its, j
    REAL dgg, fp, gam, gg, g(NMAX), h(NMAX), xi (NMAX)
    REAL FDEF(*), D(*), S(*), B(*)
    fp=func(p, FDEF, D, S, B)
    call dfunc(p, xi, FDEF, D, S, B)
    do 11 j=1, n
        g(j)=- xi (j)
        h(j)=g(j)
        xi (j)=h(j)
11  continue
    do 14 its=1, ITMAX
        iter=its
        PRINT*, ' SUBROUTINE FRPRMN/NUMBER OF ITERATIONS : ', ITER
        PRINT*, ' '
        call linmi n(p, xi, n, fret, FDEF, D, S, B)
        if(2. *abs(fret- fp). le. ftol *(abs(fret)+abs(fp)+EPS)) return
        fp=func(p, FDEF, D, S, B)
        call dfunc(p, xi, FDEF, D, S, B)
        gg=0.
        dgg=0.
        do 12 j=1, n
            gg=gg+g(j)**2
C            dgg=dgg+xi (j)**2
            dgg=dgg+(xi (j)+g(j))*xi (j)
12  continue
        if(gg. eq. 0.) return
        gam=dgg/gg
        do 13 j=1, n
            g(j)=- xi (j)
            h(j)=g(j)+gam*h(j)
            xi (j)=h(j)
13  continue
14  continue
    pause ' frprmn maximum iterations exceeded'

```

```

    return
    END
C (C) Copr. 1986- 92 Numerical Recipes Software %-, .
C
C *****
C
    SUBROUTINE l i n m i n(p, xi , n, fret, FDEF, D, S, B)
    INTEGER n, NMAX
    REAL fret, p(n), xi (n), TOL
    PARAMETER (NMAX=500, TOL=1. e- 6)
CU  USES brent, f1di m, mbrak
    INTEGER j, ncom
    REAL ax, bx, fa, fb, fx, xmi n, xx, pcom(NMAX) , xi com(NMAX) , brent
    REAL FDEF(*), D(*), S(*), B(*)
    COMMON /f1com/ pcom, xi com, ncom
    EXTERNAL f1di m
    ncom=n
    do 11 j=1, n
        pcom(j)=p(j)
        xi com(j)=xi (j)
11  continue
    ax=0.
    xx=1.
    call mbrak(ax, xx, bx, fa, fx, fb, f1di m, FDEF, D, S, B)
    fret=brent(ax, xx, bx, f1di m, TOL, xmi n, FDEF, D, S, B)
    do 12 j=1, n
        xi (j)=xmi n*xi (j)
        p(j)=p(j)+xi (j)
12  continue
    return
    END
C (C) Copr. 1986- 92 Numerical Recipes Software %-, .
C
C *****
C
    SUBROUTINE mbrak(ax, bx, cx, fa, fb, fc, func, FDEF, D, S, B)
    REAL ax, bx, cx, fa, fb, fc, func, GOLD, GLIMIT, TINY
    EXTERNAL func
    PARAMETER (GOLD=1. 618034, GLIMIT=100. , TINY=1. e- 20)

```

```

REAL dum, fu, q, r, u, ulim
REAL FDEF(*), D(*), S(*), B(*)
fa=func(ax, FDEF, D, S, B)
fb=func(bx, FDEF, D, S, B)
if(fb. gt. fa) then
    dum=ax
    ax=bx
    bx=dum
    dum=fb
    fb=fa
    fa=dum
endif
cx=bx+GOLD*(bx- ax)
fc=func(cx, FDEF, D, S, B)
1 if(fb. ge. fc) then
    r=(bx- ax) * (fb- fc)
    q=(bx- cx) * (fb- fa)
    u=bx- ((bx- cx) *q- (bx- ax) *r) / (2. *sign(max(abs(q- r), TINY), q- r))
    ulim=bx+GLIMIT*(cx- bx)
    if((bx- u) * (u- cx) . gt. 0.) then
        fu=func(u, FDEF, D, S, B)
        if(fu. lt. fc) then
            ax=bx
            fa=fb
            bx=u
            fb=fu
            return
        else if(fu. gt. fb) then
            cx=u
            fc=fu
            return
        endif
        u=cx+GOLD*(cx- bx)
        fu=func(u, FDEF, D, S, B)
    else if((cx- u) * (u- ulim) . gt. 0.) then
        fu=func(u, FDEF, D, S, B)
        if(fu. lt. fc) then
            bx=cx
            cx=u

```

```

        u=cx+GOLD*(cx-bx)
        fb=fc
        fc=fu
        fu=func(u, FDEF, D, S, B)
    endi f
else if((u-ulim)*(ulim-cx).ge.0.) then
    u=ulim
    fu=func(u, FDEF, D, S, B)
else
    u=cx+GOLD*(cx-bx)
    fu=func(u, FDEF, D, S, B)
endi f
ax=bx
bx=cx
cx=u
fa=fb
fb=fc
fc=fu
goto 1
endi f
return
END

```

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C

C

C

```

FUNCTION brent(ax, bx, cx, f, tol, xmi n, FDEF, DD, S, BB)
INTEGER ITMAX
REAL brent, ax, bx, cx, tol, xmi n, f, CGOLD, ZEPS
EXTERNAL f
PARAMETER (ITMAX=100, CGOLD=. 3819660, ZEPS=1. 0e- 10)
INTEGER iter
REAL a, b, d, e, etemp, fu, fv, fw, fx, p, q, r, tol 1, tol 2, u, v, w, x, xm
REAL FDEF(*), DD(*), S(*), BB(*)
a=mi n(ax, cx)
b=ma x(ax, cx)
v=bx
w=v
x=v

```

```

e=0.
fx=f(x, FDEF, DD, S, BB)
fv=fx
fw=fx
do 11 iter=1, ITMAX
  xm=0.5*(a+b)
  tol1=tol*abs(x)+ZEPS
  tol2=2.*tol1
  if(abs(x-xm).le.(tol2-.5*(b-a))) goto 3
  if(abs(e).gt.tol1) then
    r=(x-w)*(fx-fv)
    q=(x-v)*(fx-fw)
    p=(x-v)*q-(x-w)*r
    q=2.*(q-r)
    if(q.gt.0.) p=-p
    q=abs(q)
    etemp=e
    e=d
    if(abs(p).ge.abs(.5*q*etemp).or.p.le.q*(a-x).or.p.ge.q*(b-x))
*goto 1
    d=p/q
    u=x+d
    if(u-a.lt.tol2.or.b-u.lt.tol2) d=sign(tol1,xm-x)
    goto 2
  endif
1  if(x.ge.xm) then
    e=a-x
  else
    e=b-x
  endif
  d=CGOLD*e
2  if(abs(d).ge.tol1) then
    u=x+d
  else
    u=x+sign(tol1,d)
  endif
  fu=f(u, FDEF, DD, S, BB)
  if(fu.le.fx) then
    if(u.ge.x) then

```



```

        a=x
    else
        b=x
    endif
    v=w
    fv=fw
    w=x
    fw=fx
    x=u
    fx=fu
else
    if(u.lt.x) then
        a=u
    else
        b=u
    endif
    if(fu.le.fw .or. w.eq.x) then
        v=w
        fv=fw
        w=u
        fw=fu
    else if(fu.le.fv .or. v.eq.x .or. v.eq.w) then
        v=u
        fv=fu
    endif
endif
endif
11 continue
pause 'brent exceed maximum iterations'
3 xmin=x
brent=fx
return
END

```

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C

C

C

```

FUNCTION f1dim(x, FDEF, D, S, B)
INTEGER NMAX
REAL f1dim, func, x

```

```

PARAMETER (NMAX=500)
CU  USES func
    INTEGER j, ncom
    REAL pcom(NMAX), xi com(NMAX), xt(NMAX)
    REAL FDEF(*), D(*), S(*), B(*)
    COMMON /f1com/ pcom, xi com, ncom
    do 11 j=1, ncom
        xt(j)=pcom(j)+x*xi com(j)
11  continue
    f1dim=func(xt, FDEF, D, S, B)
    return
    END
C  (C) Copr. 1986-92 Numerical Recipes Software %-, .
C
C  *****
C
C  SUBROUTINE RESPONSE(B, A, X, Y, DETNUM, PE, UNITS, NSD, PPMCF)
C
C  Calculates the instrument response ("blur") matrix B.
C  DETNUM is the detector number.
C  PE is an INTEGER, the photon energy in KeV.
C
C
C  INTEGER I, II, K, L
C  INTEGER M, NX, NY, NXY, MNXY, IQ
C  INTEGER DETNUM, PE, NSD
C  REAL AA, RR, XK, XL, R, ASR
C  REAL NOP, GPD, TH, ANGC, RHO, AS, PHI0A, ANG2, ANG3
C  REAL A, PPMCF
C  PARAMETER(J=4)
C  INTEGER DET(J)
C  REAL B(MNXY), X(M), Y(M)
C  CHARACTER UNITS*10
C  COMMON/MN/M, NX, NY, MNXY, NXY
C
C  DATA DET/75230, 2670, 30860, 1030/
C
C  RHO = 1.6
C  II = 0
C  PPMCF = -1.0

```

C

```
IF (DETNUM .EQ. DET(1)) THEN
  IF (PE .EQ. 60) THEN
    AA = 2.131E-04
    NOP = 2152.15
    GPD = 0.0383
    AS = 3.968E-01
    ANG2 = 1.625E-02
    ANG3 = -1.513E-04
    II = 1
  ELSE IF (PE .EQ. 583) THEN
    AA = 0.979775E-04
    NOP = 893.60
    GPD = 0.858
    AS = 0.131596
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
  ELSE IF (PE .EQ. 911) THEN
    AA = 0.799150E-04
    NOP = 680.88
    GPD = 0.29
    AS = 0.106931
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
  ELSE IF (PE .EQ. 967) THEN
    AA = 0.776824E-04
    NOP = 656.71
    GPD = 0.2295
    AS = 0.1038
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
  ELSE IF (PE .EQ. 1000) THEN
    AA = 7.645E-05
    NOP = 642.66
    GPD = 0.00845
    AS = 1.021E-01
```

```

    ANG2 = 2.608E-03
    ANG3 = -1.408E-05
    II = 1
ELSE IF (PE .EQ. 2615) THEN
    AA = 0.463641E-04
    NOP = 358.19
    GPD = 0.998
    AS = 0.0625311
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
END IF
ELSE IF (DETNUM .EQ. DET(2)) THEN
    IF (PE .EQ. 583) THEN
        AA = 0.979775E-04
        NOP = 515.16
        GPD = 0.858
        AS = 0.131596
        ANG2 = 0.0
        ANG3 = 0.0
        II = 1
    ELSE IF (PE .EQ. 609) THEN
        AA = 9.6802E-5
        NOP = 500.42
        GPD = 0.461
        AS = 0.1292
        ANG2 = 0.0
        ANG3 = 0.0
        II = 1
    ELSE IF (PE .EQ. 662) THEN
        AA = 9.1504E-5
        NOP = 473.92
        GPD = 0.852
        AS = 0.12437
        ANG2 = 0.0
        ANG3 = 0.0
        II = 1
    ELSE IF (PE .EQ. 911) THEN
        AA = 0.799150E-04

```

```

NOP = 383.58
GPD = 0.29
AS = 0.106931
ANG2 = 0.0
ANG3 = 0.0
II = 1
ELSE IF (PE .EQ. 967) THEN
AA = 0.776824E-04
NOP = 369.01
GPD = 0.2295
AS = 0.1038
ANG2 = 0.0
ANG3 = 0.0
II = 1
ELSE IF (PE .EQ. 1001) THEN
AA = 7.645E-05
NOP = 360.44
GPD = 0.00845
AS = 1.021E-01
ANG2 = 0.0
ANG3 = 0.0
II = 1
ELSE IF (PE .EQ. 2615) THEN
AA = 0.463641E-04
NOP = 191.08
GPD = 0.998
AS = 0.0625311
ANG2 = 0.0
ANG3 = 0.0
II = 1
END IF
ELSE IF (DETNUM .EQ. DET(3)) THEN
IF (PE .EQ. 583) THEN
AA = 0.979775E-04
NOP = 836.42
GPD = 0.858
AS = 0.131596
ANG2 = 0.0
ANG3 = 0.0

```

```
    II = 1
ELSE IF (PE .EQ. 609) THEN
    AA = 9.6802E-5
    NOP = 811.86
    GPD = 0.461
    AS = 0.1292
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
ELSE IF (PE .EQ. 911) THEN
    AA = 0.799150E-04
    NOP = 620.93
    GPD = 0.29
    AS = 0.106931
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
ELSE IF (PE .EQ. 967) THEN
    AA = 0.776824E-04
    NOP = 597.57
    GPD = 0.2295
    AS = 0.1038
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
ELSE IF (PE .EQ. 1001) THEN
    AA = 7.645E-05
    NOP = 584.57
    GPD = 0.00845
    AS = 1.021E-01
    ANG2 = 0.0
    ANG3 = 0.0
    II = 1
ELSE IF (PE .EQ. 2615) THEN
    AA = 0.463641E-04
    NOP = 312.17
    GPD = 0.998
    AS = 0.0625311
    ANG2 = 0.0
```

```

        ANG3 = 0.0
        II = 1
    END IF
ELSE IF (DETNUM .EQ. DET(4)) THEN
    IF (PE .EQ. 662) THEN
        AA = 9.1504E-5
        NOP = 234.35
        GPD = 0.852
        AS = 0.12437
        ANG2 = 0.0
        ANG3 = 0.0
        II = 1
    END IF
ELSE
    PRINT*, ' '
    PRINT*, ' THE NUMBER OF THE DETECTOR DOES NOT MATCH ANY OF THE '
    PRINT*, ' DETECTOR NUMBERS LISTED IN THE SUBROUTINE RESPONSE. '
    PRINT*, ' '
    PRINT*, ' PLEASE CHECK YOUR INPUT FILE. '
    STOP
END IF

```

C

```

IF (II .EQ. 0) THEN
    PRINT*, ' '
    PRINT*, ' THE PHOTON ENERGY DOES NOT MATCH ANY OF THE ENERGIES '
    PRINT*, ' LISTED FOR THIS DETECTOR IN THE SUBROUTINE RESPONSE. '
    PRINT*, ' '
    PRINT*, ' PLEASE CHECK YOUR INPUT FILE. '
    STOP
END IF

```

C

```

IF (NSD .EQ. 1) THEN
    PRINT*, ' CALCULATING RESPONSE MATRIX / UNIFORM DISTRIBUTION'
    PRINT*, ' '
    PRINT*, ' ENTER YOUR CHOICE OF UNITS FOR THE SPECIFIC ACTIVITY, '
    PRINT*, ' 0 FOR Bq/g      '
    PRINT*, ' 1 FOR pCi /g     '
    PRINT*, ' 2 FOR ppm      : '
    READ*, IQ

```

```

UNITS = ' Bq/g'
IF (IQ .EQ. 1) THEN
  UNITS = ' pCi /g'
  NOP = NOP/27.03
ELSE IF (IQ .EQ. 2) THEN
  PRINT*, ' ENTER CONVERSION FACTOR (FROM pCi /g TO ppm) FOR THE'
  PRINT*, ' RADIONUCLIDE BEING MEASURED: '
  READ*, PPMCF
  UNITS = ' ppm'
  NOP = (NOP/27.03)/PPMCF
END IF

```

C

```

DO 15 I=1, M
  DO 10 K=1, NX
    DO 5 L=1, NY
      XK = A*(FLOAT(K))
      XL = A*(FLOAT(L))
      RR = ((X(I) - XK)**2) + ((Y(I) - XL)**2) + 10000.0
      R = SQRT(RR)
      ASR = AS*R
      TH = 57.30*ACOS(100.0/R)
      ANG2 = 1.0 + ANG2*TH + ANG3*(TH**2)
      PHI0A = (A**2)*RHO*(100.0/R)/12.566
      IF (ASR .GT. 10.0) THEN
        PHI0A = PHI0A*(1.0/R)*(EXPZE2Z(ASR))
      ELSE
        PHI0A = PHI0A*(EXP(ASR)/R)*(EXPINT(2, ASR))
      END IF
      B(NXY*(I-1)+NY*(K-1)+L) = EXP(-AA*R)*PHI0A*NOP*GPD*ANGC
5      CONTINUE
10     CONTINUE
15     CONTINUE
ELSE IF (NSD .EQ. 2) THEN
  PRINT*, ' CALCULATING RESPONSE MATRIX - PLANE DISTRIBUTION'
  PRINT*, ' '
  PRINT*, ' ENTER YOUR CHOICE OF UNITS FOR THE SPECIFIC ACTIVITY, '
  PRINT*, ' 0 FOR Bq/m**2 '
  PRINT*, ' 1 FOR pCi /m**2 : '
  READ*, IQ

```



```

        UNITS = ' Bq/m**2'
        IF (IQ .EQ. 1) THEN
            UNITS = ' pCi /m**2'
            NOP = NOP/27.03
        END IF

C
        DO 35 I=1, M
            DO 30 K=1, NX
                DO 25 L=1, NY
                    XK = A*(FLOAT(K))
                    XL = A*(FLOAT(L))
                    RR = ((X(I) - XK)**2) + ((Y(I) - XL)**2) + 10000.0
                    R = SQRT(RR)
                    TH = 57.30*ACOS(100.0/R)
                    ANG1 = 1.0 + ANG2*TH + ANG3*(TH**2)
                    PHI0A = ((A**2)/10000.0)/(12.566*RR)
                    B(NXY*(I-1)+NY*(K-1)+L) = EXP(-AA*R)*PHI0A*NOP*GPD*ANG1
25                CONTINUE
30                CONTINUE
35                CONTINUE
            ELSE
                PRINT*, ' '
                PRINT*, ' THE SOURCE DISTRIBUTION ENTERED DOES NOT MATCH ANY'
                PRINT*, ' OF THE DISTRIBUTIONS LISTED FOR THIS DETECTOR'
                PRINT*, ' IN THE SUBROUTINE RESPONSE. '
                PRINT*, ' '
                STOP
            END IF

C
        RETURN
    END

C
C *****
C
FUNCTION expint(n, x)
    INTEGER n, MAXIT
    REAL expint, x, EPS, FPMIN, EULER
    PARAMETER (MAXIT=100, EPS=1. e- 7, FPMIN=1. e- 30, EULER=. 5772156649)
    INTEGER i, ii, nml

```

```

REAL a, b, c, d, del, fact, h, psi
nm1=n- 1
if(n.lt. 0. or. x.lt. 0. . or. (x. eq. 0. . and. (n. eq. 0. or. n. eq. 1))) then
  pause 'bad arguments in expint'
else if(n. eq. 0) then
  expint=exp(- x) /x
else if(x. eq. 0.) then
  expint=1. /nm1
else if(x. gt. 1.) then
  b=x+n
  c=1. /FPMIN
  d=1. /b
  h=d
  do 11 i=1, MAXIT
    a=- i * (nm1+i)
    b=b+2.
    d=1. / (a*d+b)
    c=b+a/c
    del =c*d
    h=h*del
    if(abs(del - 1.) .lt. EPS) then
      expint=h*exp(- x)
      return
    endif
11  continue
  pause 'continued fraction failed in expint'
else
  if(nm1. ne. 0) then
    expint=1. /nm1
  else
    expint=- log(x) - EULER
  endif
  fact=1.
  do 13 i=1, MAXIT
    fact=- fact*x/i
    if(i. ne. nm1) then
      del =- fact/(i - nm1)
    else
      psi =- EULER

```

```

do 12 ii=1, nm1
  psi=psi+1./ii
12  continue
  del=fact*(-log(x)+psi)
  endi f
  expint=expint+del
  if(abs(del).lt.abs(expint)*EPS) return
13  continue
  pause 'series failed in expint'
endi f
return
END

```

C (C) Copr. 1986- 92 Numerical Recipes Software %-, .

C

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C

FUNCTION EXPZE2Z(X)

C

C

Function to calculate $\exp(x) * E2(x)$, where $E2(x)$ is an exponential integral. Uses the rational approximation by Paul Verbeeck, "Rational approximations for exponential integrals $E_n(x)$ ", in Bulletin de l'Academie royale de Belgique (Classe de Sciences), page 1064, 5e Serie - Tome LVI, 1970.

C

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$\exp(x) * E2(x) = (1/x) * (v1(z)/v2(z))$ where $v1, v2$ are third order polynomials, $z=1/x$, $x>1$, and stated maximum error < 9.E-06.

C

REAL X, A2, A3, B2, B3

C

A2 = 2.6731571

A3 = 0.1212641

B2 = 4.6660820

B3 = 3.7340708

C

EXPZE2Z = (1.0+(A2/X)+(A3/(X**2)))/(X*(1.0+(B2/X)+(B3/(X**2))))

C

RETURN

END

C

C
