UN01167
*WPO 49486

CALCULATING THE UNSATURATED HYDRAULIC CONDUCTIVITY WITH A NEW CLOSED-FORM ANALYTICAL MODEL

by

Rien van Genuchten*
Water Resources Program
Department of Civil Engineering
Princeton University
Princeton, New Jersey 08540

PROPERTY WMT LIBRARY

* Present address: U. S. Salinity Laboratory, USDA, 4500 Glenwood Drive, Riverside, California 92501.

78-WR-08

September 1978

Information Only

SWCF-A: 1.4.3.3: NQ: REFS

ABSTRACT

A new and relatively simple equation for the soil moisture content-pressure head curve, $\theta(h)$, is described in this report. The particular form of the equation enables one to derive closed-form analytical expressions for the relative hydraulic conductivity, K_r, when substituted in the predictive conductivity models of Burdine (1953) or Mualem (1976a). The resulting expressions for $K_r(h)$ contain three independent parameters which may be obtained by fitting the proposed soil moisture retention model to experimental data. Two different methods of curve-fitting are discussed in the report, a simple but effective graphical method, and a least-squares method requiring computer assistance. An existing non-linear least-squares curve-fitting program was modified for this purpose and is included in an appendix, together with detailed instructions regarding its use.

Results obtained with the closed form analytical expressions based on the Mualem theory were compared with observed relative hydraulic conductivity data for five soils with a wide range in hydraulic properties. The relative hydraulic conductivity was predicted well in four out of five cases. It was found that a reasonable description of the soil moisture retention curve at low moisture contents is necessary if an accurate prediction of the hydraulic conductivity is to be made.

Information Only

ACKNOWLEDGEMENT

This work was supported, in part, by funds obtained from the Solid and Hazardous Waste Research Division, U. S. Environmental Protection Agency, Municipal Environmental Research Laboratory, Cincinnati, Ohio (EPA Grant No. R803827-01).

Information Only

TABLE OF CONTENTS

Information Only

LIST OF FIGURES

Information Only

Fig. 8. Comparison of observed (open circles) and calculated curves (solid lines) of the relative hydraulic conductivity of Silt Loam G.E.3. The predicted curves were obtained for three different values of the residual moisture content, θ_r: 0.05 (curve a), 0.10 (curve b), and 0.15 cm^3/cm^3 (curve c).

Fig. 9. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Hygiene Sandstone. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Fig. 10. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Touchet Silt Loam G.E.3. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Fig. 11. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Silt Loam G.E.3. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Fig. 12. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Beit Netofa Clay. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Fig. 13. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Beit Netofa Clay. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve. The last four data points of the observed soil moisture retention curve were not considered in the fitting process.

Fig. 14. Observed (circles) and calculated curves (solid lines) of the soil hydraulic properties of Guelph Loam. The drying and wetting branches of the relative hydraulic conductivity curve were predicted from knowledge of the curve-fitted branches of the soil moisture retention curve.

Information Only

LIST OF TABLES

Information Only

INTRODUCTION

The use of numerical models for simulating fluid flow and mass transport in the unsaturated zone has become increasingly popular the last few years. Recent literature indeed demonstrates that much effort is put into the development of such models using both finite difference (Bresler, 1975; Amerman, 1976) and finite element techniques (Reeves and Duguid, 1975; Segol, 1976). Unfortunately, it appears that the ability to fully characterize the simulated system has not kept pace with the numerical and modeling expertise. Probably the single most important factor limiting the successful application of unsaturated flow theory to actual field problems is the lack of information regarding the parameters entering the governing transport equations. Reliable estimates of the unsaturated hydraulic conductivity are especially difficult to obtain, partly because of its extensive variability in the field, and partly because measuring this parameter is time-consuming and expensive. Several investigators have, for these reasons, used models for calculating the unsaturated conductivity from the more easily measured soil moisture retention curve. Very popular among these models has been the Millington-Quirk method (Millington and Quirk, 1961), various forms of which have been applied with some success in a number of studies (cf. Jackson *et al.*, 1965; Jackson, 1972; Green and Corey, 1971; Bruce, 1972). Unfortunately, this method also has the disadvantage of producing tabular results which, for example when applied to nonhomogeneous soils in multi-dimensional unsaturated flow models, are quite tedious to use.

Closed-form analytical expressions for predicting the unsaturated hydraulic conductivity have also been developed. For example, Brooks and

Information Only

Corey (1964) and Jeppson (1974) each used an analytical expression for the conductivity based on the Burdine theory (Burdine, 1953). Brooks and Corey (1964, 1966) obtained fairly accurate predictions with their equations, even though a discontinuity is present in the slope of both the moisture retention curve and the unsaturated hydraulic conductivity curve at some negative value of the pressure head (this point is often referred to as the bubbling pressure). Such a discontinuity sometimes prevents rapid convergence in numerical saturated-unsaturated flow problems. It also appears that predictions based on the Brooks and Corey equations are somewhat less accurate than those obtained with various forms of the (modified) Millington-Quirk method.

Recently Mualem (1976a) derived a new model for predicting the hydraulic conductivity from knowledge of the soil moisture retention curve and the conductivity at saturation. Mualem's derivation leads to a simple integral formula for the unsaturated hydraulic conductivity which enables one to derive closed-form analytical expressions, provided suitable equations for the soil moisture retention curves are available. It is the purpose of this report to derive such closed-form analytical expressions. The theories of both Mualem and Burdine are used for this derivation. The resulting conductivity models generally contain three independent parameters which may be obtained from the soil moisture retention data by means of curve-fitting. Two different methods of curve-fitting are discussed in this paper, a simple graphical method which enables one to obtain the parameters without requiring computer assistance, and a more elaborate non-linear least-squares curve-fitting method requiring the assistance of a digital computer. An existing computer model was modified for this purpose and is included in the appendix. Results

Information Only

obtained with the closed-form equations based on the Mualem theory are compared with observed data for a few soils having widely varying hydraulic properties.

Information Only

MATHEMATICAL DEVELOPMENT

The following equation was derived by Mualem (1976a) for predicting the relative hydraulic conductivity (K_r) from knowledge of the soil moisture retention curve

$$K_r = \Theta^{\frac{1}{2}} \left[\int_0^{\Theta} \frac{1}{h(x)}\, dx \Big/ \int_0^{1} \frac{1}{h(x)}\, dx \right]^2 \qquad (1)$$

where $h=h(\Theta)$ is the pressure head, given here as a function of the dimensionless moisture content:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}. \qquad (2)$$

In this equation, s and r indicate saturated and residual values of the soil moisture content (θ), respectively. To solve (1), an expression relating the dimensionless moisture content to the pressure head is needed. An attractive class of $\Theta(h)$-functions, adopted in this study, is given by the following general equation

$$\Theta = \left[\frac{1}{1+(\alpha h)^n} \right]^m \qquad (3)$$

where α, n and m are as yet undetermined parameters. To simplify notation later, h in (3) is assumed to be positive. Equation (3) with m=1 has been successfully used in many studies to describe soil moisture retention data (Ahuja and Schwartzendruber, 1972; Endelman *et al.*, 1974;

Information Only

Haverkamp *et al.*, 1977). A typical $\theta(h)$-curve based on Eq. (2) and (3) is shown in Fig. 1. Note that a nearly symmetrical "S"-shaped curve is obtained, and that the slope $(d\theta/dh)$ becomes zero when the moisture content approaches both its saturated and residual values.

Simple, closed-form expressions for $K_r(\Theta)$ can be derived when certain restrictions are imposed upon the values of m and n allowed in (3). Solving this equation for $h=h(\Theta)$ and substituting the resulting expression into (1) gives

$$K_r(\Theta) = \Theta^{\frac{1}{2}} \left[\frac{f(\Theta)}{f(1)}\right]^2 \qquad (4)$$

where $f(\Theta)$ is given by

$$f(\Theta) = \int_0^{\Theta} \left[\frac{x^{1/m}}{1-x^{1/m}}\right]^{1/n} dx. \qquad (5)$$

Substitution of $x=y^m$ into (5) leads to

$$f(\Theta) = m \int_0^{\Theta^{1/m}} y^{m-1+1/n} \, (1-y)^{-1/n} \, dy. \qquad (6)$$

Equation (6) represents a particular form of the Incomplete Beta-function (see for example Abramowitz and Stegun, 1970; p. 944) and, in its most general case, no closed-form expression can be derived. However, it is easily shown that for integer values of $k=m-1+1/n$ the integration can be carried out without difficulties. For the particular case when $k=0$ (i.e. $m=1-1/n$) integration of (6) yields

$$f(\Theta) = 1-(1-\Theta^{1/m})^m \qquad (m=1-1/n) \qquad (7)$$

Information Only

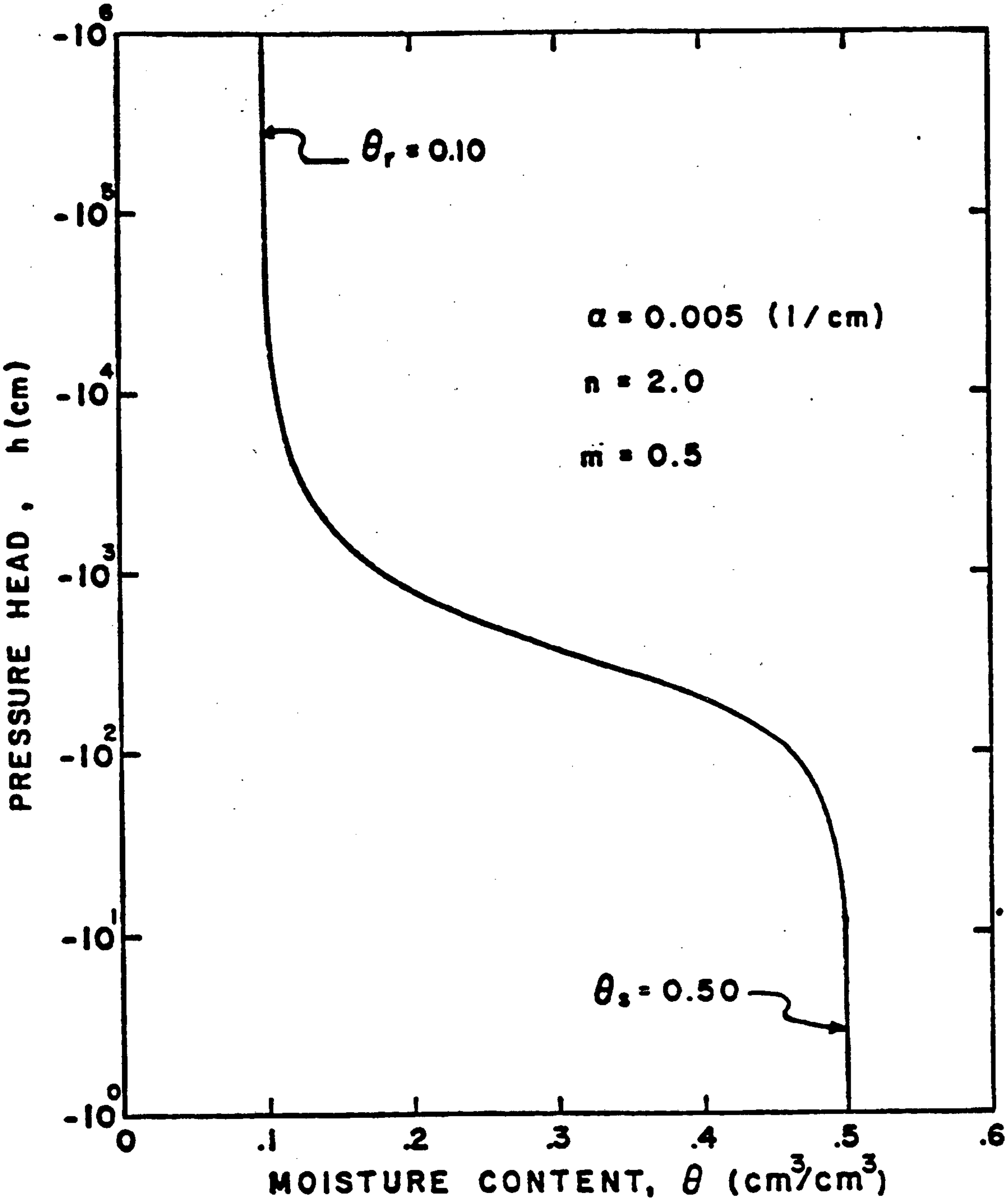

Fig. 1. Typical plot of the soil moisture retention curve based on Eq. (3).

Information Only

and, because $f(1) = 1$, (4) becomes

$$K_r(\Theta) = \Theta^{\frac{1}{2}}\left[1-(1-\Theta^{1/m})^m\right]^2 . \qquad \begin{matrix}(m=1-1/n)\\(0<m<1)\end{matrix} \qquad (8)$$

The relative hydraulic conductivity may also be expressed in terms of the pressure head by substituting (3) into (8), i.e.

$$K_r(h) = \frac{\{1-(\alpha h)^{n-1}\,[1+(\alpha h)^n]^{-m}\}^2}{[1+(\alpha h)^n]^{m/2}} . \qquad (m=1-1/n) \qquad (9)$$

From the hydraulic conductivity and the soil moisture retention curve one may also derive an expression for the soil moisture diffusivity, which is defined as

$$D(\theta) = K(\theta)\left|\frac{dh}{d\theta}\right| . \qquad (10)$$

This leads to the following equation for $D(\Theta)$:

$$D(\Theta) = \frac{(1-m)K_s}{\alpha m(\theta_s-\theta_r)}\,\Theta^{\frac{1}{2}-1/m}\left[(1-\Theta^{1/m})^{-m} + (1-\Theta^{1/m})^m - 2\right] \qquad (11)$$

where K_s is the hydraulic conductivity at saturation. Equations (9) and (11) are shown graphically in Fig. 2 and 3, respectively, using the same values of α, n and m(=1-1/n) as in Fig. 1. As can be seen from Fig. 2, the relative hydraulic conductivity starts out with a slope of zero at pressure head values near zero, but then falls off increasingly rapid as h decreases. The soil moisture diffusivity, on the other hand, attains (as does the soil moisture retention curve) a fairly symmetrical "S"-shaped curve with infinite gradients, $d(\log D)/d\theta$, when θ approach-

Information Only

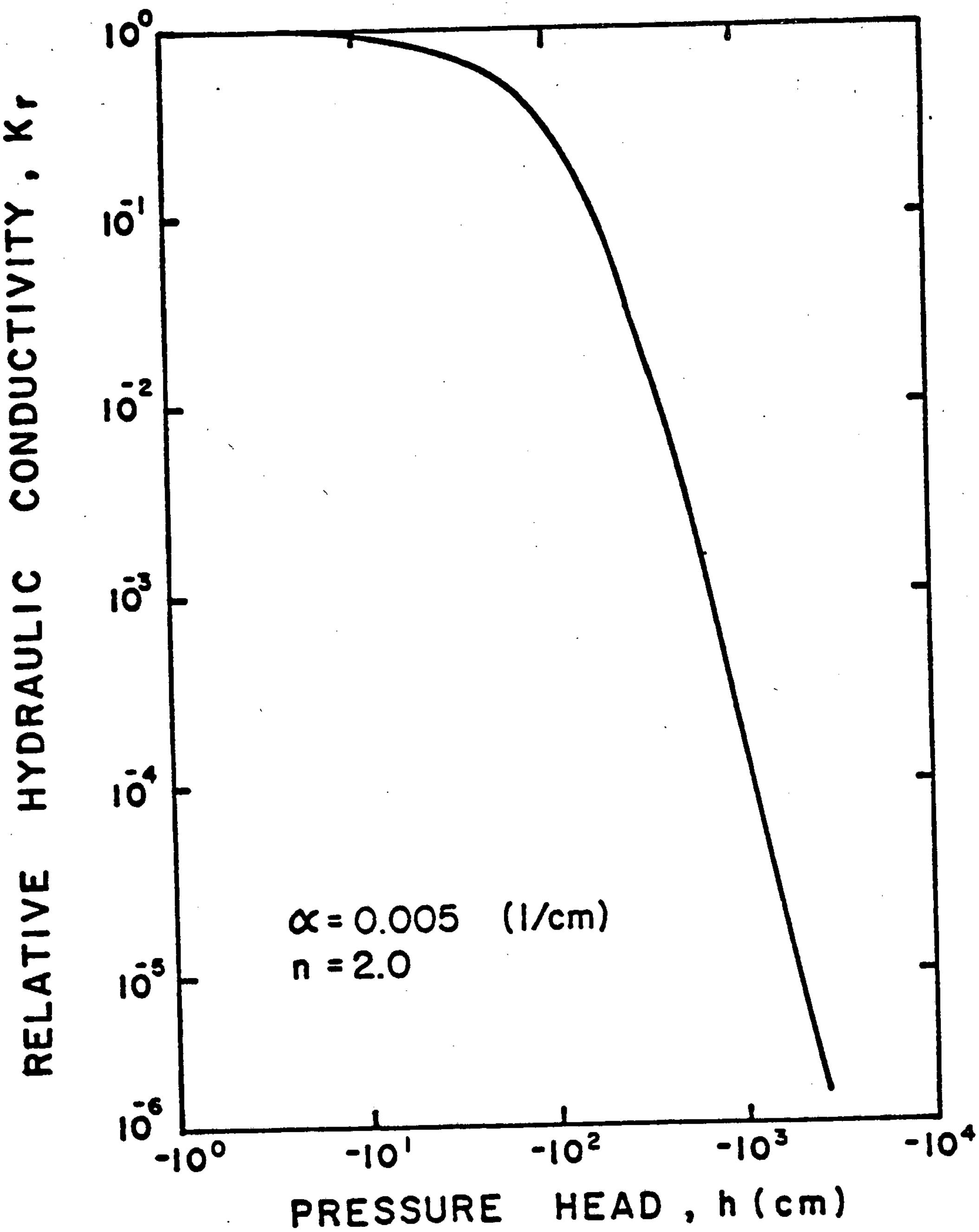

Fig. 2. Plot of the relative hydraulic conductivity versus pressure head as predicted from knowledge of the soil moisture retention curve shown in Fig. 1.

Information Only

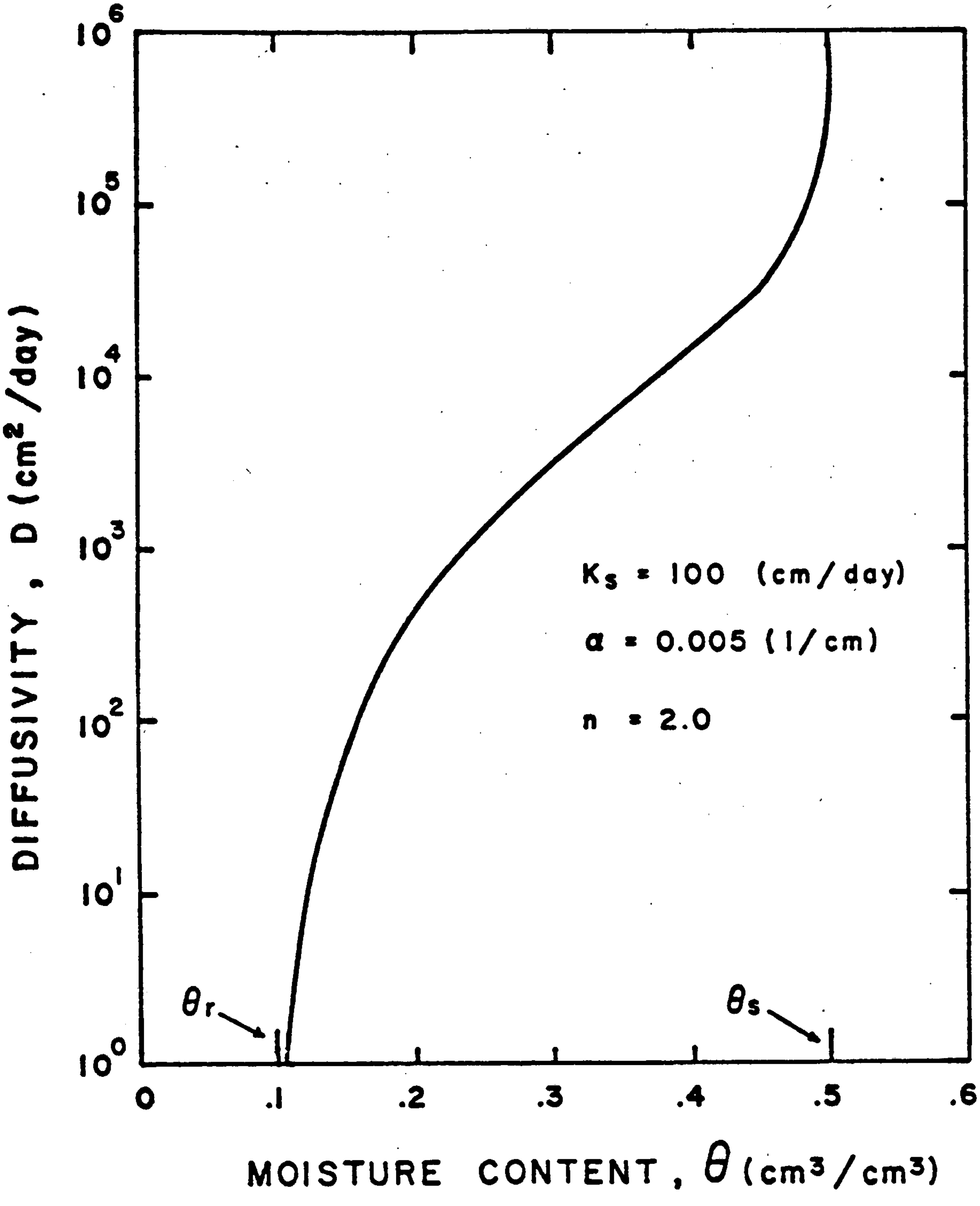

Fig. 3. Plot of the soil moisture diffusivity versus moisture content as predicted from knowledge of the soil moisture retention curve shown in Fig. 1, and the hydraulic conductivity at saturation.

Information Only

es either θ_r or θ_s. Note that the diffusivity becomes infinite when θ approaches θ_s. Only at intermediate values of the moisture content (approximately between θ=0.25 and θ=0.45 in Fig. 3) does the diffusivity acquire the often assumed exponential dependency on the moisture content. Similar features of the soil moisture diffusivity were obtained and discussed by Ahuja and Schwartzendruber (1972), using the following special form of $D(\theta)$:

$$D(\theta) = \frac{a\,\theta^p}{(\theta_s-\theta)^q} \tag{12}$$

where a, p and q are material characteristic parameters.

The soil hydraulic properties derived above were obtained by assuming that $k=m-1+1/n=0$ in (6). One may also derive closed-form expressions for other integer values of k. For k=1, for example, the conductivity becomes

$$K_r(\Theta) = \Theta^{\frac{1}{2}}\left[1-m(1-\Theta^{1/m})^{m-1}+(m-1)(1-\Theta^{1/m})^{m}\right]^2. \quad (m=2-1/n) \tag{13}$$

While this particular model is not only more complicated than model (8), it also represents only a slight pertubation of the earlier function. Hence, (13) does not present an attractive alternative for (8), and will not be discussed further.

Similar results as above for the Mualem theory may also be obtained when the Burdine theory is taken as a point of departure. The equation given by Burdine (1953) is:

$$K_r(\Theta) = \Theta^2 \int_0^{\Theta} \frac{1}{h^2(x)}\,dx \Bigg/ \int_0^1 \frac{1}{h^2(x)}\,dx\,. \tag{14}$$

Information Only

The analysis proceeds in a similar way as before. Equation (3) is inverted to give $h=h(\Theta)$ and substitution of the resulting expression into (14) yields

$$K_r(\Theta) = \Theta^2 f(\Theta)/f(1) \tag{15}$$

where

$$f(\Theta) = \int_0^\Theta \left[\frac{x^{1/m}}{1-x^{1/m}}\right]^{2/n} dx \,. \tag{16}$$

Substituting $x=y^m$ into (16) gives

$$f(\Theta) = m \int_0^{\Theta^{1/m}} y^{m-1+2/n}(1-y)^{-2/n}\, dy. \tag{17}$$

Again it is assumed that the exponent of y in (17) vanishes. Hence $m=1-2/n$, and (17) reduces to

$$f(\Theta) = 1 - (1-\Theta^{1/m})^m \,. \tag{18}$$

The relative hydraulic conductivity hence becomes

$$K_r(\Theta) = \Theta^2 \left[1-(1-\Theta^{1/m})^m\right] \qquad \begin{array}{l}(m=1-2/n)\\ (0<m<1;\ n>2)\end{array} \tag{19}$$

or in terms of the pressure head

$$K_r(h) = \frac{1-(\alpha h)^{n-2}\left[1+(\alpha h)^n\right]^{-m}}{\left[1+(\alpha h)^n\right]^{2m}} \,. \tag{20}$$

Information Only

The soil moisture diffusivity for this case is given by

$$D(\Theta) = \frac{(1-m)K_s}{2\alpha m(\theta_s-\theta_r)} \Theta^{(3-1/m)/2} [(1-\Theta^{1/m})^{-(m+1)/2} - (1-\Theta^{1/m})^{(m-1)/2}]. \tag{21}$$

Preliminary tests indicated that (8) generated results that were, in most cases, in better agreement with experimental data than (19). Through an extensive series of comparisons, also Mualem (1976a) concluded that predictions based on his theory (i.e., based directly on Eq. (1) by means of numerical approximations) were generally more accurate than those based on various forms of the Burdine theory (including the Millington-Quirk method). It is not the intent of this paper to give accuracy comparisons between various closed-form analytical conductivity expressions. Only a brief discussion of the equations derived by Brooks and Corey (1964) will be given here, since their model of the soil moisture retention curve represents a limiting case of the moisture retention model discussed in this study.

Brooks and Corey (1964; 1966) concluded from comparisons with a large number of experimental data that the soil moisture retention curve $\Theta(h)$ could be described reasonably well with the following general equation

$$\Theta = (h/h_b)^{-\lambda} \qquad (h<h_b) \tag{22}$$

where h_b is the bubbling pressure (approximately equal to the air entry value), and λ a soil characteristic parameter. Comparing Eq. (22) and (3), one sees that (3) reduces to (22) for large values of the pressure head, i.e.

Information Only

$$\Theta = (\alpha h)^{-mn} \quad . \tag{23}$$

For the Mualem theory one has $m=1-1/n$, and hence $\lambda=n-1$, while for the Burdine theory ($m=1-2/n$) one finds that $\lambda=n-2$. The parameter α, furthermore, is inversely related to the bubbling pressure, h_b. Brooks and Corey used the Burdine theory to predict the relative hydraulic conductivity and the soil moisture diffusivity. They derived the following expressions

$$K_r(\Theta) = \Theta^{3+2/\lambda} \tag{24a}$$

$$K_r(h) = (\alpha h)^{-2-3\lambda} \tag{24b}$$

$$D(\Theta) = \frac{K_s}{\alpha\lambda(\theta_s-\theta_r)}\Theta^{2+1/\lambda} \quad . \tag{25}$$

Through substitution of (22) into (1), similar equations can be obtained when the Mualem theory is used:

$$K_r(\Theta) = \Theta^{5/2+2/\lambda} \tag{26a}$$

$$K_r(h) = (\alpha h)^{-2-5\lambda/2} \tag{26b}$$

$$D(\Theta) = \frac{K_s}{\alpha\lambda(\theta_s-\theta_r)}\Theta^{3/2+1/\lambda} \quad . \tag{27}$$

Figure 4 compares the different expressions given above with the earlier obtained relations for the conductivity and the diffusivity [Eq. (3), (9), and (11)]. The parameters α and n were chosen to be the same as before (i.e., $\alpha=0.005$ and $n=2$), while λ was assumed to be equal to $(n-1)$.

Information Only

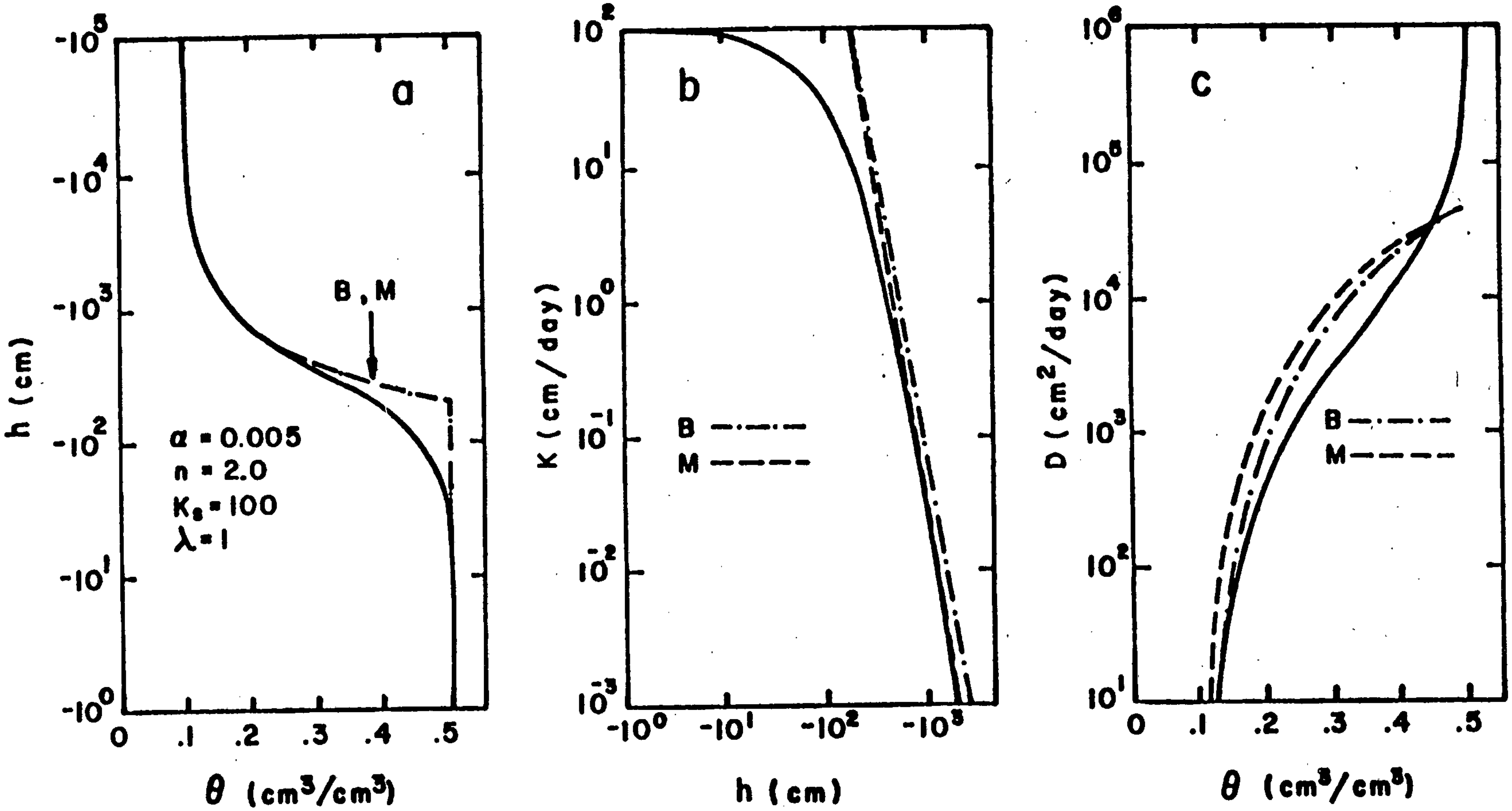

Fig. 4. Comparison of the proposed soil hydraulic functions (solid lines) with curves obtained by applying either the Mualem theory (M; dashed lines) or the Burdine theory (B; dashed-dotted lines) to the Brooks and Corey model of the soil moisture retention curve.

Information Only

The soil moisture retention curves for all three cases become then identical for sufficiently low values of the moisture content. Figure 4a shows that the Brooks and Corey model of the $\theta(h)$-curve approaches the curve based on (3) asymptotically when θ decreases. However, large deviations between the two models occur then θ approaches its saturated value. The curves based on (22) reach θ_s at a much lower value of h, i.e. at -200 cm ($h=h_b=1/\alpha$). The most important deviations between the predicted conductivity curves are also present at or near the bubbling pressure (Fig. 4b). As expected, the curves based on Eq. (9) and (26b) (the solid and dashed lines, respectively) approach each other asymptotically when h becomes increasingly negative, while the curve used by Brooks and Corey (the dashed-dotted line) remains somewhat separated from the other two because of the different exponent in the conductivity equation [see Eq. (24b) and (26b)]. The diffusivity curves (Fig. 4c) show their most important differences at both the intermediate and higher values of the moisture content. Note that the diffusivity curves based on (22) remain finite (D_s=50,000 cm^2/day) when θ approaches θ_s, while the solid line (Eq. 10) goes to infinity at saturation. It should be emphasized that Fig. 4 was included only to demonstrate typical properties of the various conductivity and diffusivity models, and that the figure should not be viewed as an accuracy evaluation of any one model.

Information Only

The soil moisture content (θ) as a function of the pressure head (h) is given by Eq. (2) and (3), i.e.,

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha h)^n]^m} \qquad (28)$$

where, as before, it is understood that h is positive, and where for the Mualem model

$$m = 1 - 1/n. \qquad (29)$$

Equation (28) contains four independent parameters (θ_r, θ_s, α, and n), which have to be estimated from observed soil moisture retention data. Of these four, the saturated moisture content (θ_s) is probably always available as it is easily obtained experimentally. Also the residual moisture content (θ_r) may be measured experimentally, for example by determining the moisture content on very dry soil. Unfortunately, θ_r-measurements are not always made routinely, and hence have to be estimated by extrapolating existing soil moisture retention data. Assuming for the moment that sufficiently accurate estimates of both θ_r and θ_s are available, the following procedure can then be used to obtain estimates of the remaining parameters α and n.

Differentiation of (28) gives

$$\frac{d\theta}{dh} = \frac{-\alpha m(\theta_s - \theta_r)}{1-m} \Theta^{1/m}(1-\Theta^{1/m})^m \qquad (30)$$

Information Only

where the right-hand side is expressed in terms of Θ, rather than h. The pressure head may also be expressed in terms of the moisture content by inverting (3), i.e.,

$$h = \frac{1}{\alpha} (\Theta^{-1/m} - 1)^{1/n} . \tag{31}$$

Elimination of α from (29) and (31) results in

$$h \frac{d\theta}{dh} = \frac{-m(\theta_s - \theta_r)}{1-m} \Theta (1-\Theta^{1/m}). \tag{32}$$

The right-hand side of this equation contains only the unknown parameter m (both θ_s and θ_r are assumed to be known). Hence it is possible to obtain estimates of m by determining the product of the slope ($d\theta/dh$) and the pressure head (h) at some point on the $\theta(h)$-curve. Soil moisture retention data are often plotted on a semi-logarithmic scale. One may take advantage of this fact by noting that

$$\frac{d\theta}{d(\log h)} = (\ln 10)\, h \frac{d\theta}{dh} . \tag{33}$$

Let S be the absolute value of the slope of Θ with respect to log h, i.e.,

$$S = \left| \frac{d\Theta}{d(\log h)} \right| \tag{34a}$$

or, equivalently,

$$S = \frac{1}{(\theta_s - \theta_r)} \left| \frac{d\theta}{d(\log h)} \right| . \tag{34b}$$

Information Only

Combining (32), (33), and (34b) leads to the following expression for S

$$S = 2.303 \frac{m}{1-m} \Theta (1-\Theta^{1/m}). \tag{35}$$

The best location on the $\theta(h)$ curve for evaluating the slope S is about halfway between θ_r and θ_s. Let P be the point on the soil moisture retention curve for which $\Theta=\frac{1}{2}$ (see Fig. 5). From Eq. (2) and (31) it follows then that the coordinates of P are given by

$$\theta_P = (\theta_s+\theta_r)/2 \tag{36a}$$

$$h_P = \frac{1}{\alpha} (2^{1/m}-1)^{1-m} \tag{36b}$$

while Eq. (35) reduces to

$$S_P(m) = 1.151 \frac{m}{1-m} (1-2^{-1/m}). \tag{37a}$$

The subscript P in these equations is used to indicate evaluation at P. Equation (37a) can also be expressed in terms of n

$$S_P(n) = 1.151 (n-1)(1-2^{n/(1-n)}). \tag{37b}$$

Figure 6 gives a plot of S_P as a function of both n and m. This figure may be used to obtain an estimate of n once the slope S_P is determined graphically from the experimental data. For relatively large values of n, (37b) is closely approximated by

Information Only

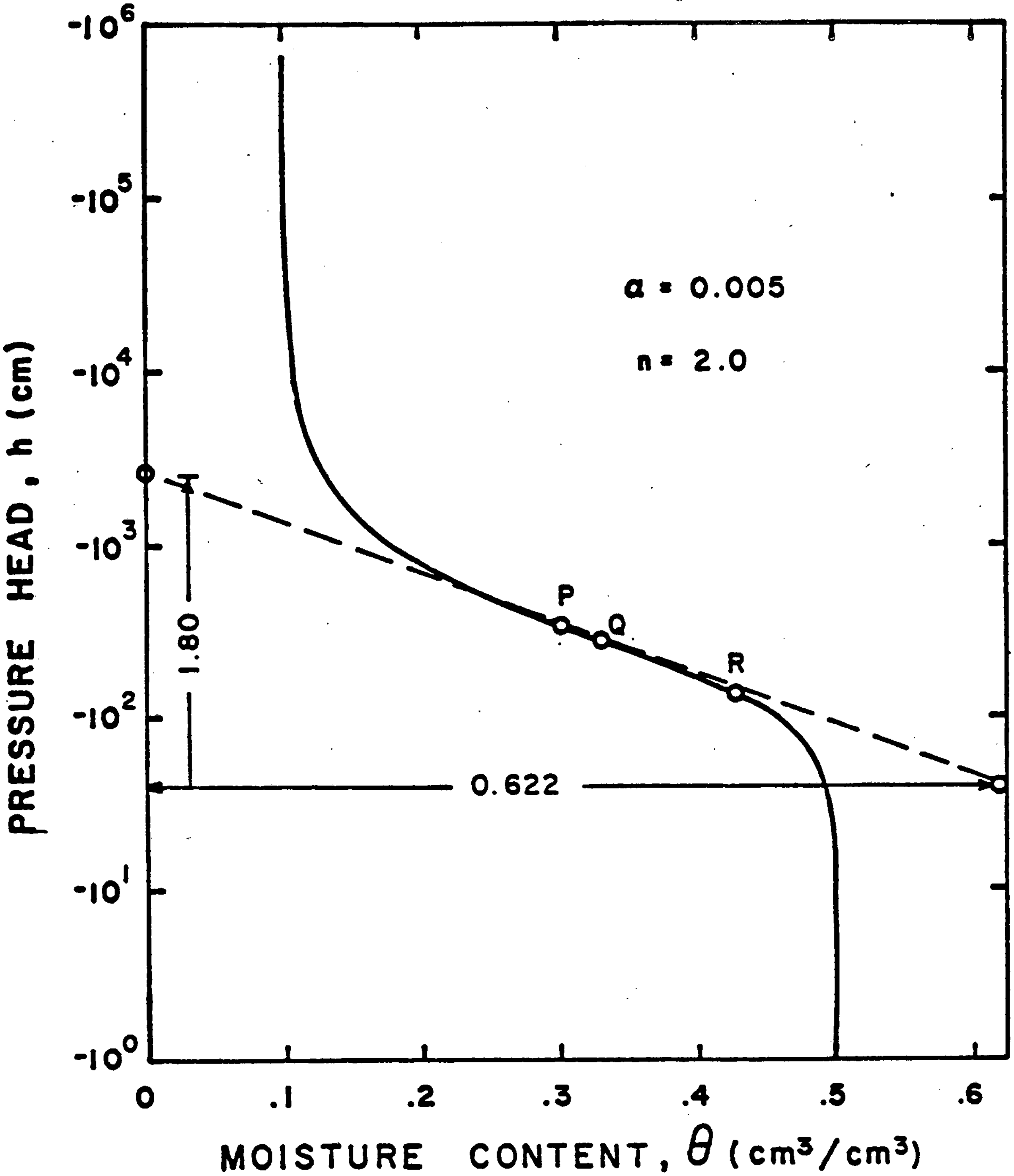

Fig. 5. Plot showing the location of the points P, Q, and R on the soil moisture retention curve. The point P is situated halfway between θ_r (=0.10) and θ_s (=0.50), the point Q represents the inflection point of the curve (semilogarithmic plot), while R represents the inflection point if the curve were plotted on a normal (θ versus h) scale.

Information Only

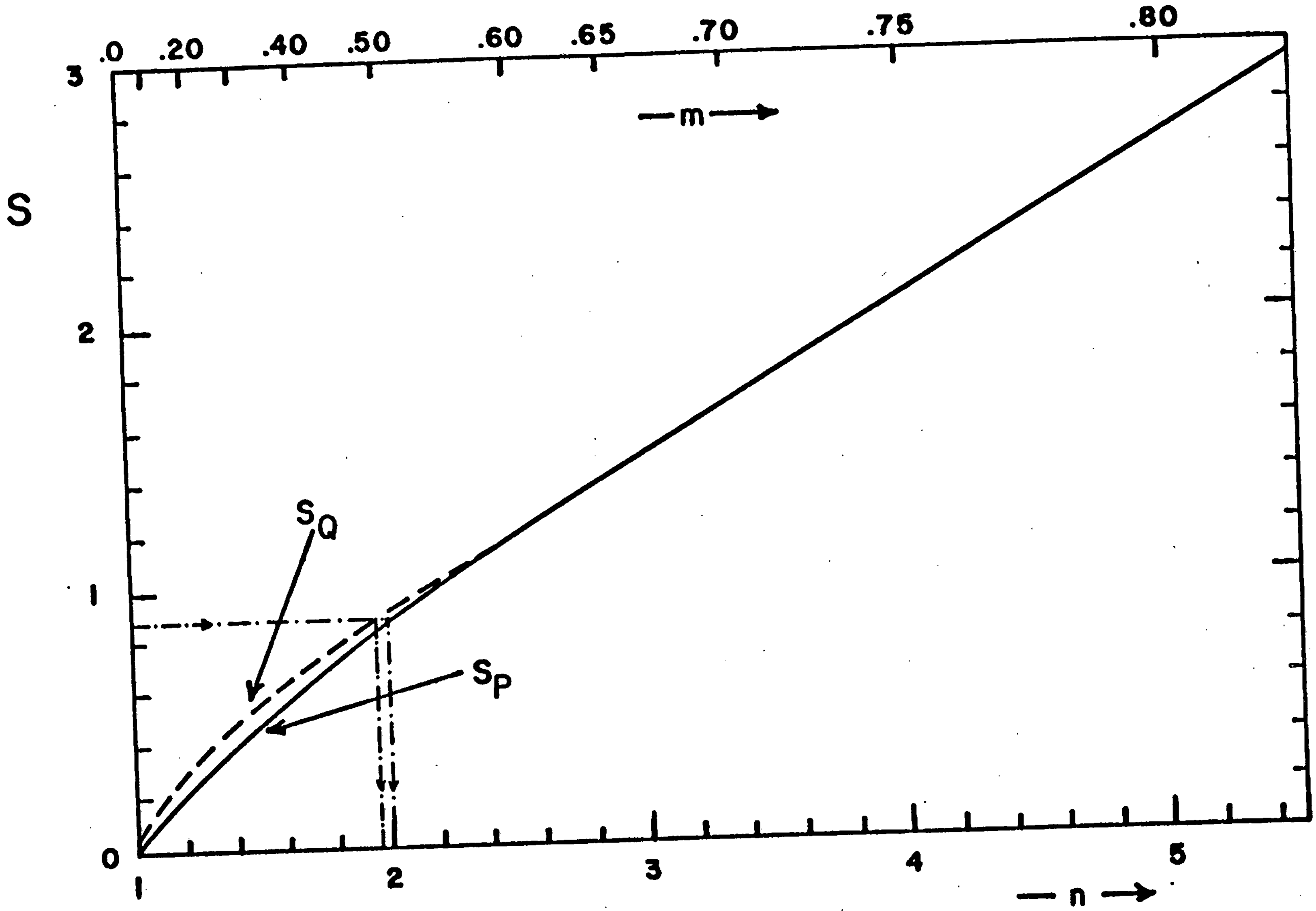

Fig. 6. Plots of the dimensionless slopes S_P and S_Q as functions of the parameters n and m (=1-1/n).

Information Only

$$S_p(n) = 0.5769\ n - 0.211 \qquad (n>4) \tag{37c}$$

from which one obtains

$$n = 1.733\ S_p + 0.37. \qquad (n>4) \tag{38}$$

Alternatively, n can also be obtained from (37b) itself by rearranging the equation into the following iterative scheme:

$$n = 1 + 0.869\ S_p/(1-2^{n/(1-n)}). \tag{39}$$

The iterative solution converges rapidly. Even for a wild initial guess of n generally only two or three iterations are necessary to obtain answers correct to within 1%. Once n (or m) is determined, α can be evaluated with (36b).

An alternative approach for estimating n and α from experimental data follows by considering the inflection point on the θ versus log h curve (the point marked "Q" in Fig. 5). Here one has

$$\frac{d^2\Theta}{d(\log h)^2} = 0\ . \tag{40}$$

Calculation of the inflection point is greatly simplified by noting that

$$\frac{d^2\Theta}{d(\log h)^2} = (\ln 10)^2\ (h^2 \frac{d^2\Theta}{dh^2} + h \frac{d\Theta}{dh}). \tag{41}$$

It is easily verified that substitution of (3) into (40) and subsequent expansion leads to

Information Only

$$\Theta_Q = \left[\frac{m}{1+m}\right]^m . \quad (42)$$

Hence, the coordinates of the inflection point are

$$\theta_Q = \theta_r + (\theta_s - \theta_r) \left[\frac{m}{1+m}\right]^m \quad . \quad (43a)$$

$$h_Q = \frac{1}{\alpha} \ m^{m-1} . \quad (43b)$$

From (43a) it follows that, at least theoretically, one could estimate the value of m directly by locating the inflection point on the soil moisture retention curve. However, from Fig. 5 it is clear that it is not easy to determine this point accurately (even less so when the curve is based on experimental data). It seems, therefore, better to again estimate m from the slope of the curve. Substitution of (42) into (35) gives

$$S_Q(m) = \frac{2.303}{1-m} \left[\frac{m}{1+m}\right]^{m+1} \quad (44a)$$

or, in terms of n,

$$S_Q(n) = 2.303 \ n \left[\frac{n-1}{2n-1}\right]^{2-1/n} . \quad (44b)$$

Figure 6 shows that $S_P(n)$ and $S_Q(n)$ define approximately the same curve, especially for the larger n-values. This is not surprising since the points P and Q are generally very close together on the soil moisture retention curve. Fig. 5, furthermore, shows that both points define approximately the same gradient. Hence the n-values obtained from the sketched

Information Only

slope should be nearly identical.

Instead of using the graphical procedure of Fig. 6, it is also possible to obtain n as a function of S_Q by iteratively solving Eq. (44b) itself. The following converging scheme was used for that purpose:

$$n = \frac{3}{4} + (n - \frac{1}{2}) A + \frac{1}{4(1-2A)}, \qquad A = \left(\frac{S_Q}{2.303\ n}\right)^{\frac{n}{2n-1}} \qquad (45)$$

As an illustrative example, the foregoing procedure was applied to the curve shown in Fig. 5. Assuming the indicated slope to be the same for both points, P and Q, one obtains for S_P and S_Q (Eq. 34b):

$$S_P = S_Q = \frac{0.622}{(0.40)\ (1.8)}$$

$$= 0.864.$$

From Fig. 6, or Eq. (39) and (45), it then follows that $n_P = 2.00$ and $n_Q = 1.96$. Hence from (20) one finds $m_P = 0.50$ and $m_Q = 0.49$. From Fig. 5 it follows that $\log(h_P) = 2.54$ and $\log(h_Q) = 2.43$. Finally, from Eq. (36b) one obtains

$$\alpha_P = \frac{1}{h_P}(2^{1/m} - 1)^{1-m}$$

$$= 10^{-2.54}(2^2-1)^{0.5} = 0.0050$$

and from (43b)

Information Only

$$\alpha_Q = \frac{1}{h_Q} m^{m-1}$$

$$= 10^{-2.43}(0.49)^{-0.51} = 0.0053.$$

The relative hydraulic conductivities hence are (Eq. 8):

$$K_r(\Theta) = \Theta^{\frac{1}{2}}\left[1-(1-\Theta^{2.00})^{0.50}\right]^2 \quad \text{(based on } S_P) \qquad (46a)$$

$$K_r(\Theta) = \Theta^{\frac{1}{2}}\left[1-(1-\Theta^{2.04})^{0.49}\right]^2 \quad \text{(based on } S_Q). \qquad (46b)$$

Equation (46a) exactly reproduces the conductivity equation one would have obtained if the original data shown in Fig. 5 were used in Eq. (8). Equations (46a) and (46b) generate nearly the same curve when plotted versus or versus h. Minor differences between the curves occur only at the extreme dry side of the curves, and are caused by the fact that the same slope was used to calculate both S_P and S_Q (in reality, S_Q should have been measured somewhat larger than S_P).

The parameters α and n can also be estimated from soil moisture retention data which are plotted on a normal θ versus h scale. The procedure for finding the two parameters is similar to that used before. Equation (37) still holds provided, however, that S is calculated with Eq. (33) and (34). These two equations show that now estimates of both h and the slope, $d\theta/dh$, are necessary for evaluating S. Equations (43) and (44), on the other hand, have to be modified because the inflection point of the θ(h)-curve does not coincide with the inflection point of the θ(log h) curve. Contrary to (40), one has now

Information Only

$$\frac{d^2\theta}{dh^2} = 0. \tag{47}$$

Expansion of (47) yields the following coordinates of the inflection point on the $\theta(h)$-curve (this point is marked "R" on the $\theta(\log h)$-curve in Fig. (5).

$$\theta_R = \theta_r + (\theta_s - \theta_r)\ (1+m)^{-m} \tag{48a}$$

$$h_R = \frac{1}{\alpha}\, m^{1-m} \tag{48b}$$

and (35) becomes

$$S_R(m) = 2.303\ m(1+m)^{-(1+m)}\ . \tag{49}$$

Information Only

INFLUENCE OF THE RESIDUAL MOISTURE CONTENT

The foregoing discussion assumes that independent measurements of the saturated and residual moisture contents are available. While θ_s is usually easy to obtain by direct measurement, θ_r is often much more difficult to quantify. In fact, in many cases θ_r may become an ill-defined parameter. The residual moisture content in this report is defined as the moisture content for which the gradient ($d\theta/dh$) becomes zero (excluding the region near θ_s which has also a zero gradient). Also the hydraulic conductivity will approach zero when θ approaches θ_r. From a practical point of view it seems sufficient to define θ_r as the moisture content at some large negative value of the pressure head, e.g., at -10^{-6} cm. Even in that case, however, significant decreases in h are likely to result in further desorption of moisture. It seems that such further changes in θ are fairly unimportant for most practical field problems. In fact, they would be inconsistent with the general shape of the $\theta(h)$-curve defined by (22), and probably invalidate the concept of a residual moisture content itself. A reasonable estimate of θ_r is necessary for an accurate prediction of the hydraulic conductivity, even though its influence on the predictions is generally less than that of α and n. The following example problem demonstrates the effect of θ_r on the conductivity predictions.

Figure 7a shows the soil moisture retention curve of Silt Loam G.E.3, for values of h between zero and 10^{-3} cm. (Reisenauer, 1963). The open circles represent data points of the curve, and were taken from the catalogue of Mualem (1976b). Because only a limited portion of the the curve is defined, an accurate estimate of θ_r is not easy to obtain.

Information Only

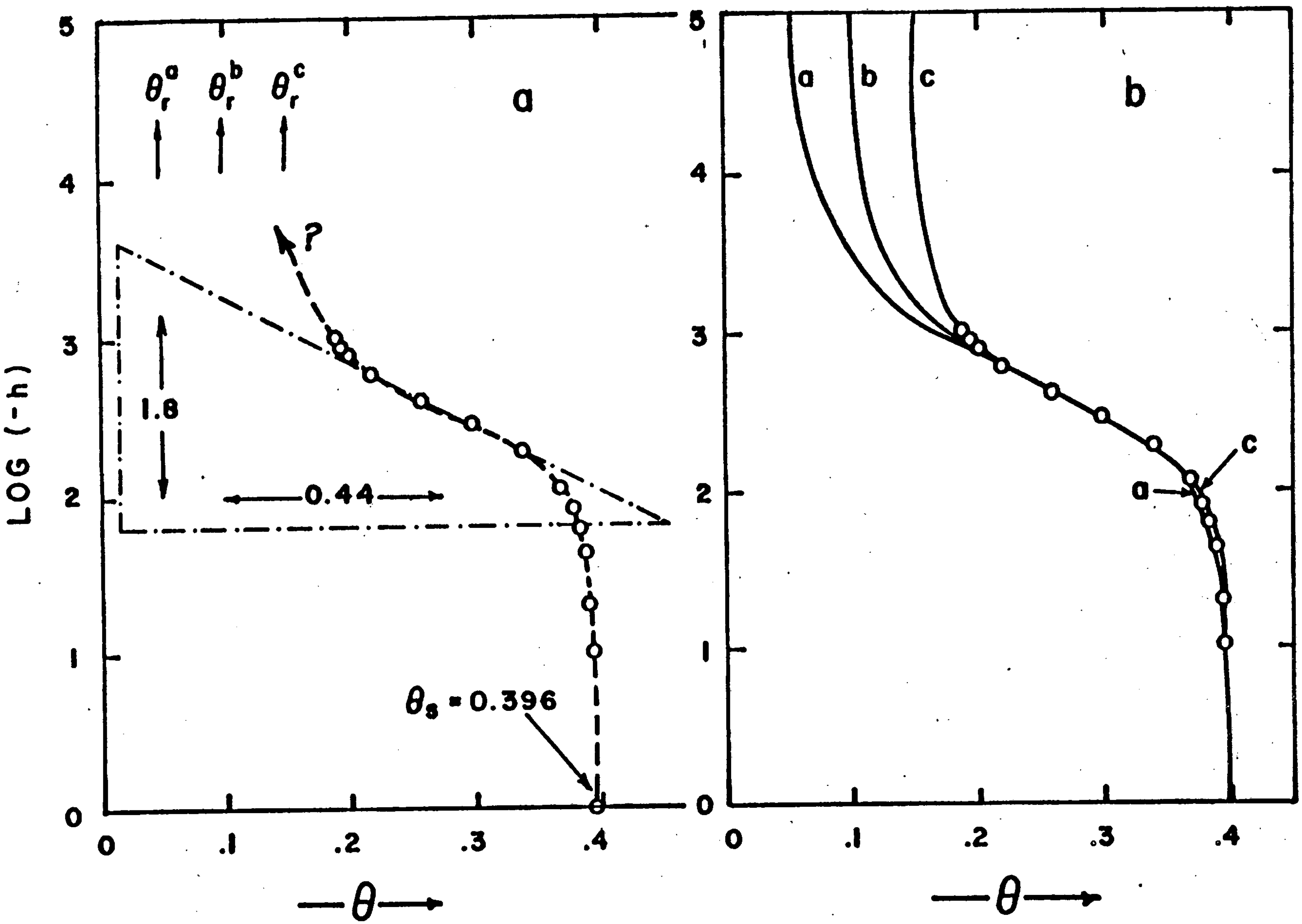

Fig. 7. Plot illustrating the graphical determination of the parameters α and n for three different values of the residual moisture content: $\theta_r^a = 0.05$ (curve a), $\theta_r^b = 0.10$ (curve b), and $\theta_r^c = 0.15$ cm^3/cm^3 (curve c). The open circles represent the observed soil moisture retention curve of Silt Loam G.E.3 (Reisenauer, 1963).

Information Only

Three different values for θ_r were chosen rather arbitrarily (0.05, 0.10, and 0.15 cm^3/cm^3, respectively), and subsequently used to calculate the hydraulic conductivity. The calculations, based on Eq. (36) and (37), are summarized in Table 1. The slope of the $\theta(\log h)$-curve at $\Theta=\frac{1}{2}$ was assumed to be the same for all three cases (step 6 in Table 1), a sufficiently accurate assumption in this case. Figure 7b compares the calculated retention curves with the experimental curve. Each of the

Table 1. Calculation of the parameters α and n from the observed soil moisture retention curve of Silt Loam G.E.3, using three different values for θ_r (θ_s=0.396)

STEP	θ_r^a	θ_r^b	θ_r^c
1. Estimate θ_r	0.050	0.100	0.150
2. Obtain $(\theta_s-\theta_r)$	0.346	0.296	0.246
3. Calculate $\theta_p=(\theta_s+\theta_r)/2$.	0.223	0.248	0.273
4. Obtain $\log(h_p)$ from data (Fig. 7a)	2.76	2.65	2.55
5. Calculate h_p	575.	447.	355.
6. Estimate $d\theta/d(\log h)$ at θ_p (Fig. 7a) (=0.44/1.8)	0.244	0.244	0.244
7. Calculate S_p [$=0.244/(\theta_s-\theta_r)$] (Eq. 34b)	0.706	0.826	0.994
8. Obtain n from Fig. 6 or Eq. (39)	1.77	1.95	2.21
9. Calculate m (=1-1/n)	0.435	0.487	0.548
10. Calculate α (Eq. 36b)	0.0038	0.0040	0.0043

three curves describes the experimental curve fairly accurately, although curve c (based on θ_r^c) fits the data points somewhat better at the dry end of the curve than the other two. On the other hand, this curve also

Information Only

slightly overpredicts the observed curve at the higher moisture contents, i.e. near h=-100 cm. The predicted conductivity curves are presented in Fig. 8. Again, all three curves give a reasonable description of the experimental points. The higher conductivity values are most accurately described by curve b, while curve c is the most accurate one at the dry side of the curve. However, it is clear that all three curves are acceptable, and hence that the influence of the residual moisture content, at least for this particular example, is not that significant.

In the above example θ_r was selected beforehand in an arbitrary way, and still no clear procedure is available for obtaining a reasonable estimate of θ_r from measured data, especially when only part of the $\theta(h)$ curve is given. To alleviate this problem, at least partially, a least-squares curve-fitting technique was used to estimate the three parameters θ_r, α, and n directly from the observed data. An existing non-linear least-squares curve-fitting program (Meeter, 1964) was modified and adapted for this purpose. The program uses the maximum neighborhood method of Marquardt (1964), which is based on an optimum interpolation between the Taylor series method and the method of steepest descent. A detailed analysis of this technique is also given by Daniel and Wood (1973). A listing of the computer program is given in Appendix A.

Information Only

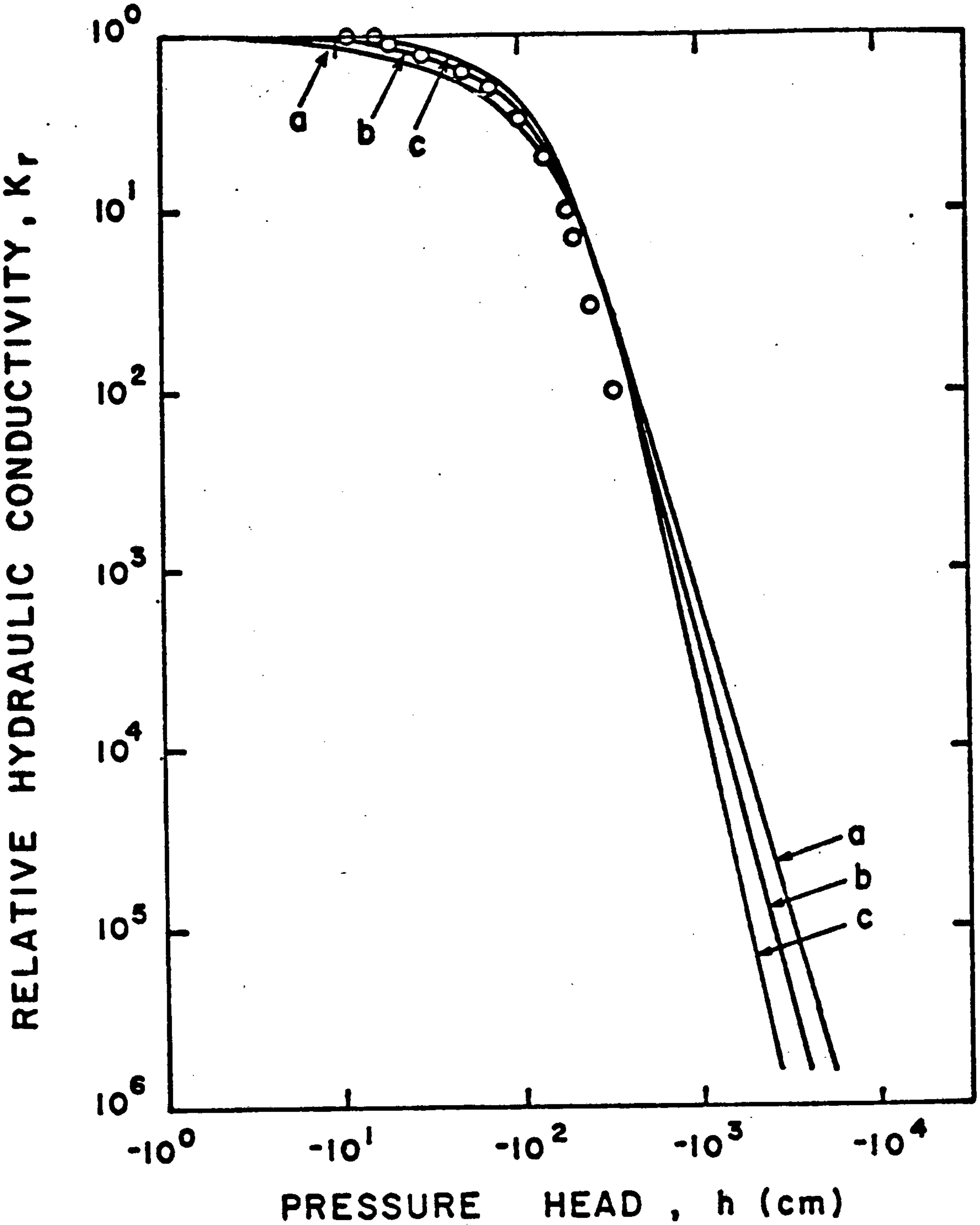

Fig. 8. Comparison of observed (open circles) and calculated curves (solid lines) of the relative hydraulic conductivity of Silt Loam G.E.3. The predicted curves were obtained for three different values of the residual moisture content, θ_r: 0.05 (curve a), 0.10 (curve b), and 0.15 cm^3/cm^3 (curve c).

Information Only

RESULTS

In this section comparisons are given between observed and calculated conductivity curves for five soils. The examples were selected for soils with widely different hydraulic properties. The observed data for each example, with the exception of the last one, were taken from the soils catalogue of Mualem (1976b). Table 2 summarizes some of the soil-physical properties of the five soils. Estimates of the parameters θ_r, α, and n are also included in the table, and were obtained by fitting Eq. (28) to the observed soil moisture retention data.

Results for Hygiene Sandstone (Brooks and Corey, 1964) are shown in Fig. 9. This soil has a rather narrow pore-size distribution, causing the soil moisture release curve to become very steep around h=-125 cm. A relatively high value of 10.4 for n was obtained for this soil, a direct consequence of the steep curve. The value of α was found to be 0.079 (1/cm), approximately the inverse of the pressure head at which the soil

Table 2. Soil-physical properties of the five example soils.

SOIL NAME	θ_s (cm^3/cm^3)	θ_r (cm^3/cm^3)	K_s (cm/day)	α (1/cm)	n (---)
Hygiene sandstone	.250	.153	108.0	.0079	10.4
Touchet Silt Loam G.E.3	.469	.190	303.0	.00505	7.9
Silt Loam G.E.3	.396	.131	4.96	.00423	2.06
Guelph Loam (drying)	.520	.218	31.6	.0115	2.03
(wetting)	(.434)	.218	-	.0200	2.76
Beit Netofa Clay	.446	.286	.082	.00202	1.59

Information Only

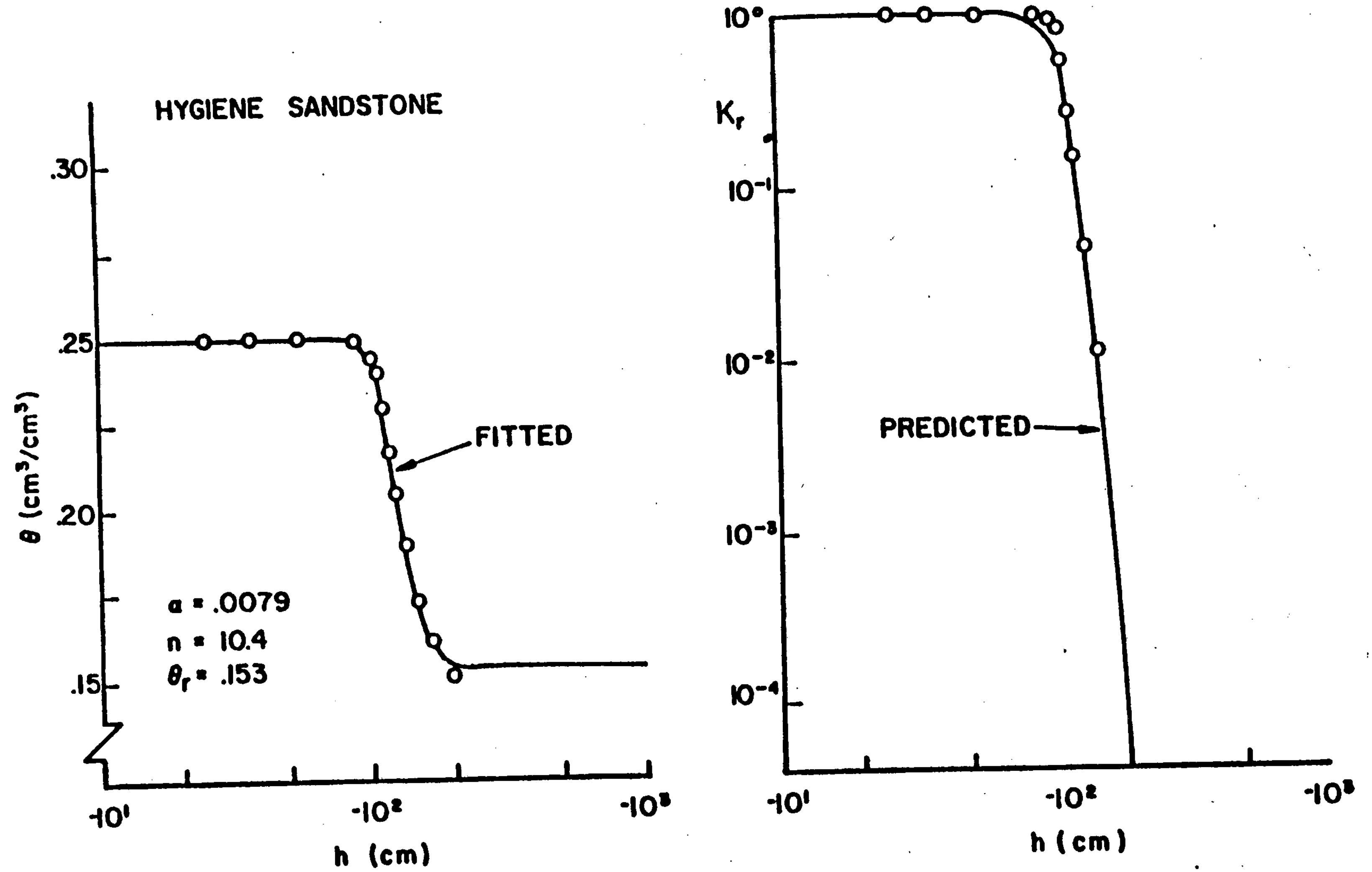

Fig. 9. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Hygiene Sandstone. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Information Only

moisture retention curve becomes the steepest (Fig. 9). This, of course, follows directly from Eq. (36b) and (43b) which, for values of m close to one (i.e., for n large), reduce to $h_P = h_Q = 1/\alpha$. In that case h_P and h_Q both become identical to the bubbling pressure, h_b, used in the Brooks and Corey equations (see Eq. 22 and 23). Fig. 9 shows a nearly exact prediction of the relative hydraulic conductivity, with only some minor deviations occurring at the higher conductivity values.

Results obtained for Touchet Silt Loam G.E.3 (Brooks and Corey, 1964), shown in Fig. 10, are very similar to those for Hygiene Sandstone. The curves in this case are also very steep (n=7.09), and again a good description of the relative hydraulic conductivity is obtained.

Figure 11 presents results obtained for Silt Loam G.E.3 (Reisenauer, 1963). This example was already discussed in the previous section, where estimates of α and n were obtained graphically for three different values of the residual moisture content. It was then found that θ_r-values of 0.10 and 0.15 gave the best answers, both for the description of the soil moisture retention curve and the relative hydraulic conductivity. Interestingly, the three-parameter curve-fitting gave a value of 0.131, approximately the average of these two θ_r-values. However, it remains clear that the value of θ_r for this particular example is poorly defined, and that a considerable change in θ_r will have only minor effects on the calculated curves. Data for this soil were also used as an illustrative example for the non-linear least-squares curve-fitting program given in Appendix A. Output of the program (see Appendix A) shows that the 95% confidence interval for θ_r is given by 0.131 ($\pm$ 16%). By comparison, these intervals are .00423 ($\pm$ 5%) and 2.06 ($\pm$ 9%) for α and n, respectively. It may be noted here that the computer

Information Only

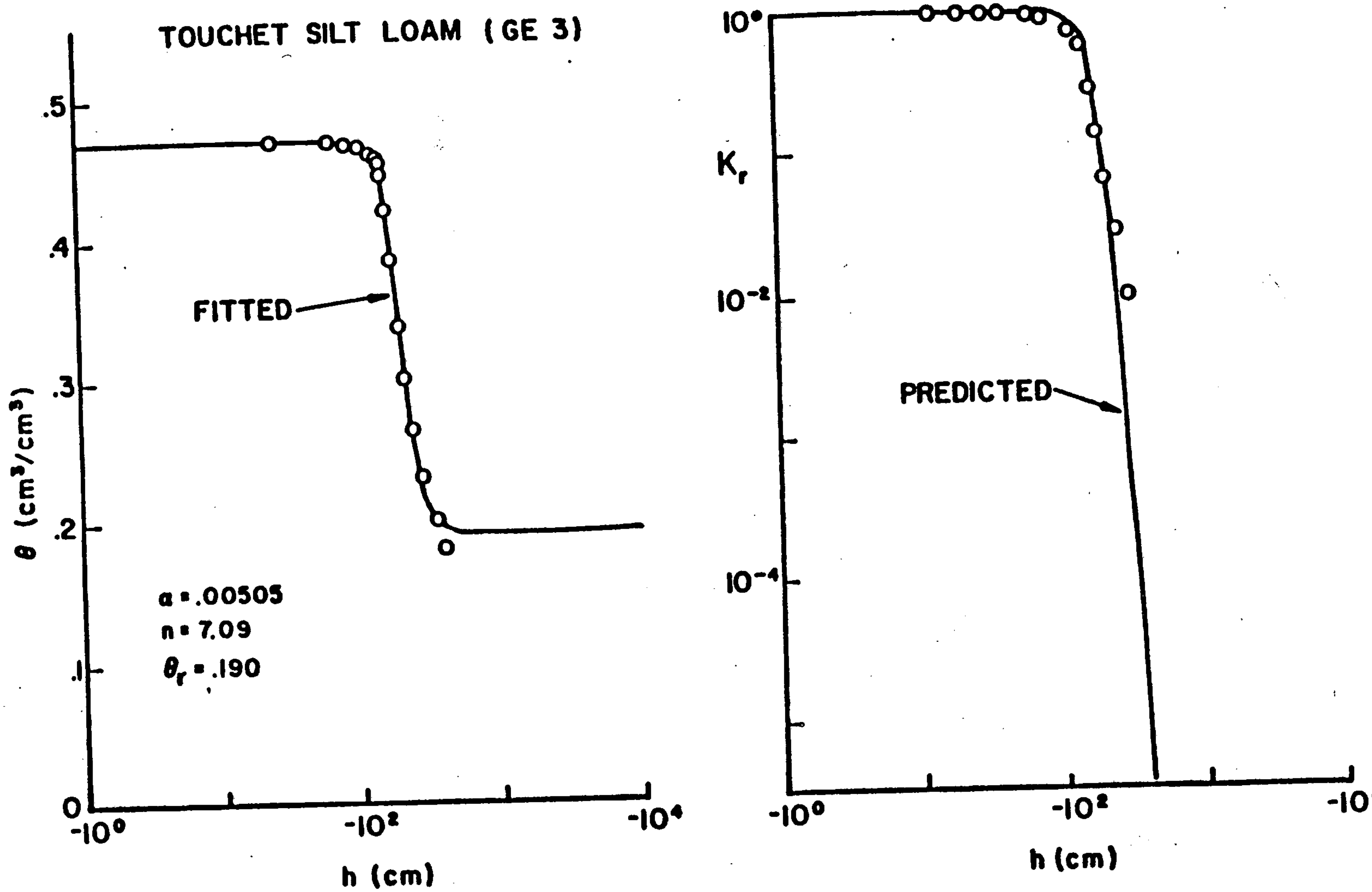

Fig. 10. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Touchet Silt Loam G.E.3. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Information Only

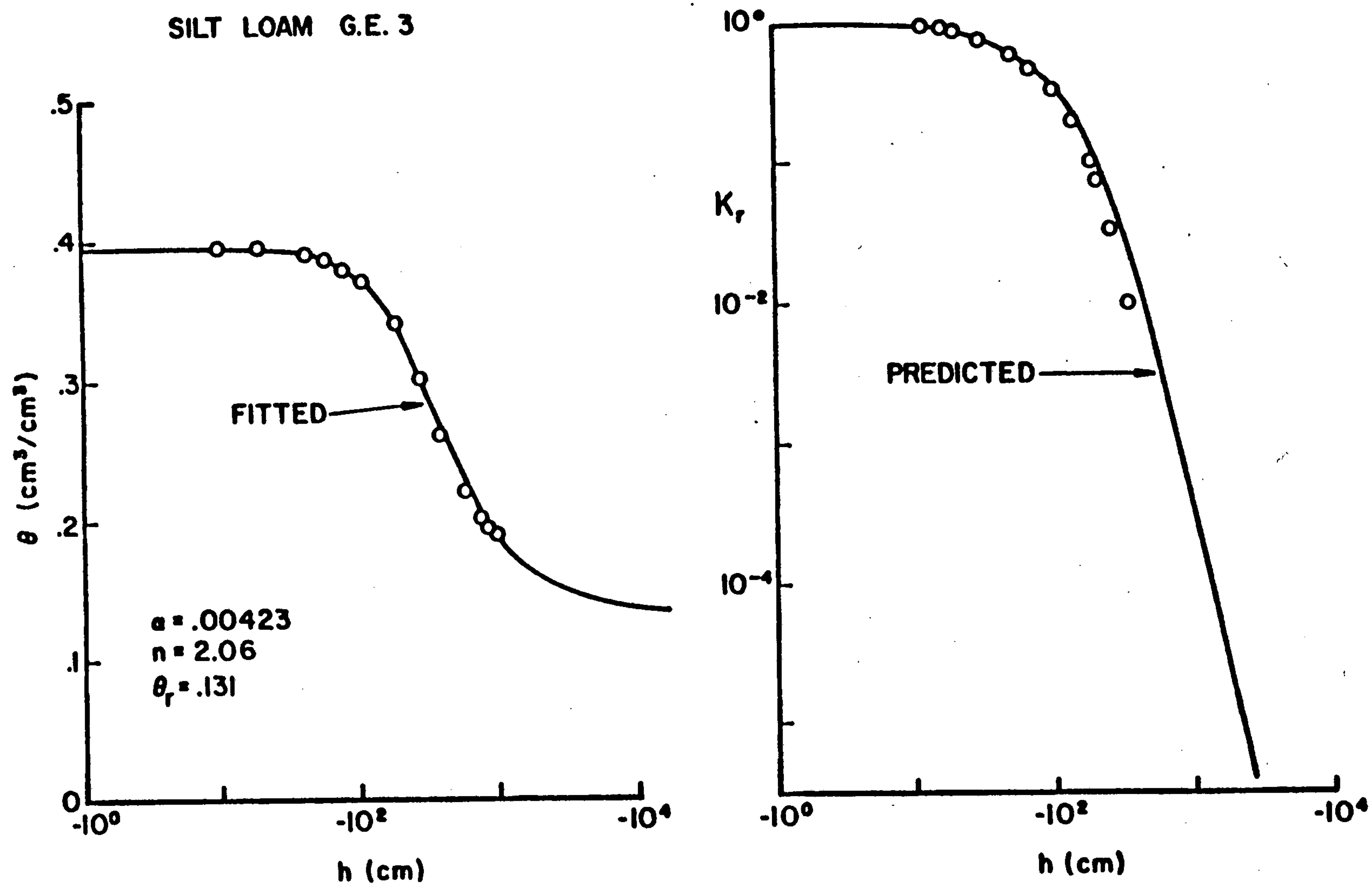

Fig. 11. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Silt Loam G.F.3. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

Information Only

program also provides for a correlation matrix between the different parameters. Results, for example, show that θ_r is highly correlated with n but much less than with α, and that α and n are nearly independent of each other. Some of these effects are also noticeable from the calculations in Table 1.

The first three examples each showed excellent agreement between observed and predicted conductivity curves. Predictions obtained for Beit Netofa Clay (Rawitz, 1965), however, were found to be much less accurate (Fig. 12). The higher conductivity values are seriously underpredicted, and also the general shape of the predicted curve is considerably different from the observed one. It seems that much of the poor predictions can be traced back to the inability of equation (28) to match the observed soil moisture retention data. For example, the residual moisture content was estimated to be zero, a rather surprising result since clay soils have generally higher θ_r-values than coarser soils (the saturated hydraulic conductivity of this soil is only 0.082 cm/day). Limited data at the lower moisture contents further increases doubt about the accuracy of the fitted θ_r-value. A careful inspection of the observed curve shows that the gradient of the curve changes fairly suddenly at approximately h=-10,000 cm (the slope suddenly becomes more negative). The location of the last four data points, in particular, appears to be inconsistent with the general shape of curves based on (28). With some imagination one could also identify an inflection point on the observed curve at a pressure head of about -2,000 cm. The observed curve should have become flatter from that point on if equation (28) were to describe the data points. Because of the seemingly unreasonable low value of θ_r, the break in the slope of the curve at h=-10,000 cm, and the presence

Information Only

Information Only

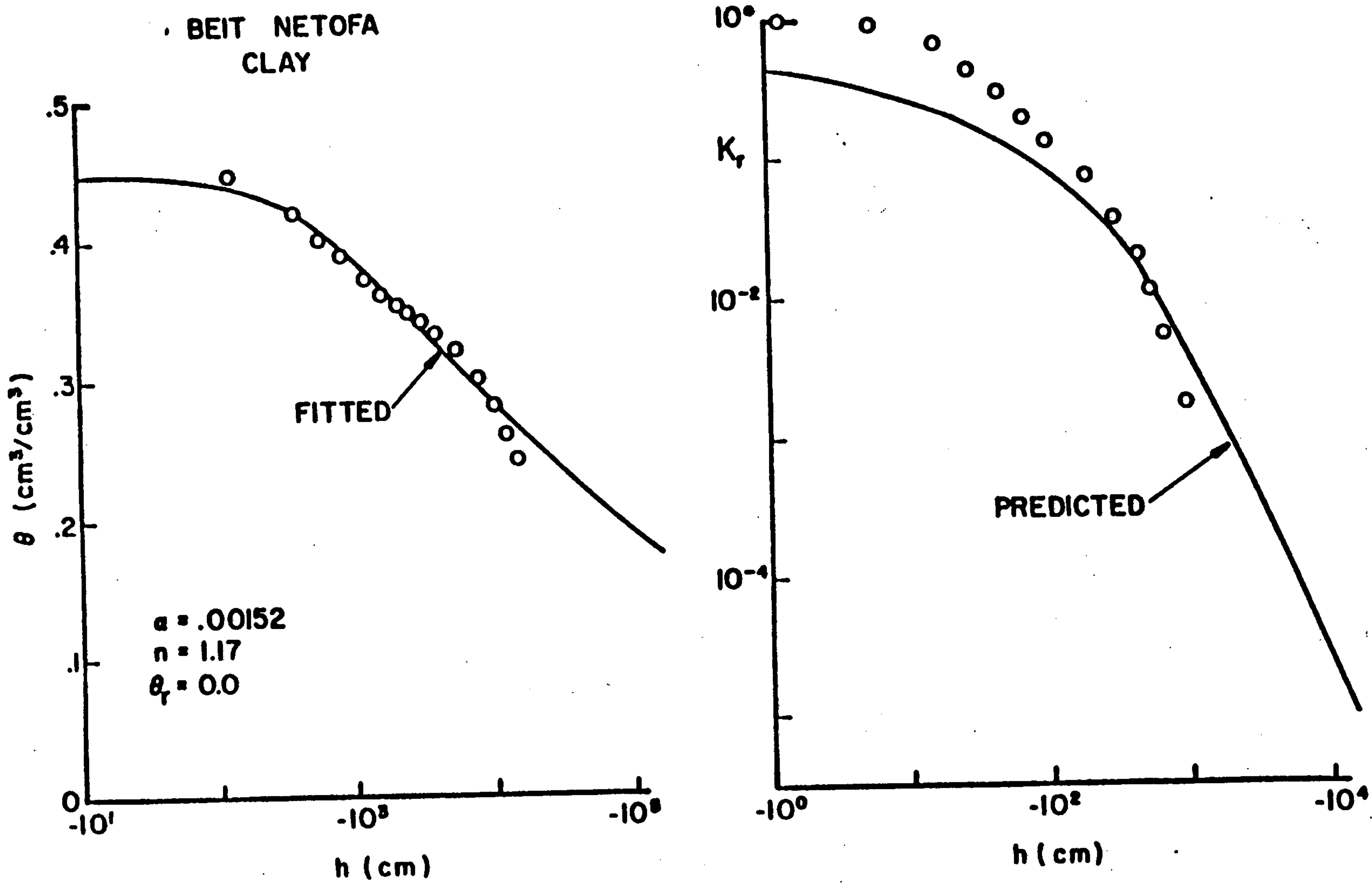

Fig. 12. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Beit Netofa Clay. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve.

of an inflection point at $h = -2,000$ cm, an attempt was made to improve the predictions by deleting rather arbitrarily the last four data points at the dry side of the curve. Fig. 13 shows that the soil moisture retention curve is now much better described (with the obvious exception of the last four data points). Also the description of the conductivity curve is improved somewhat. At least the general shape of the curve is described more accurately, even though the predicted curve is still displaced to the right of the observed one. The example shows that by deleting only four points at the dry end of the curve a completing different value of θ_r is obtained (0.286 versus 0.0 cm^3/cm^3). This case demonstrates again the importance of having some independent procedure for estimating the residual moisture content.

Results for Guelph Loam (Elrick and Bowman, 1964) are given in Fig. 14. This example represents a case in which hysteresis is present in the soil moisture retention curve. The observed data of this example were taken directly from the original study (Figs. 2 and 3 of Elrick and Bowman, 1964). For the wetting branch a maximum ("saturated") value of 0.434 for the moisture content was used, being the highest measured value. Also the wetting branch of the hydraulic conductivity curve was matched to the highest value of K_r measured during wetting (Fig. 14). The value of θ_r, furthermore, was assumed to be the same for drying and wetting, and was obtained from the drying branch of the curve. Both the drying and wetting branches of the soil moisture retention curve are adequately described by (28). Also the conductivity curves are reasonably well described, even though the predicted curves are slightly below the observed ones. Note that some hysteresis is predicted in the relative hydraulic conductivity. Although this is generally to be expected when two different

Information Only

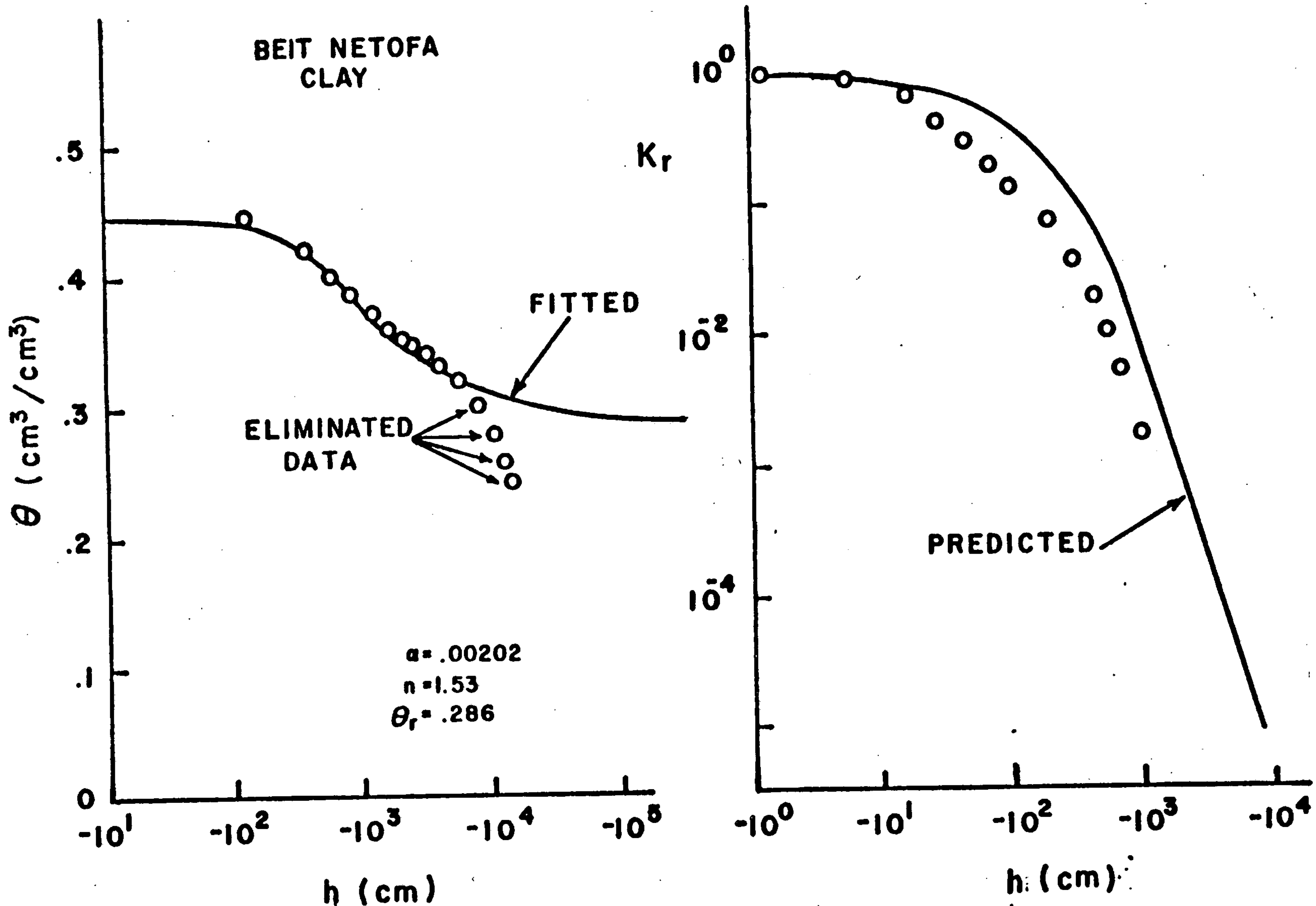

Fig. 13. Observed (open circles) and calculated curves (solid lines) of the soil hydraulic properties of Beit Netofa Clay. The relative hydraulic conductivity was predicted from knowledge of the curve-fitted soil moisture retention curve. The last four data points of the observed soil moisture retention curve were not considered in the fitting process.

Information Only

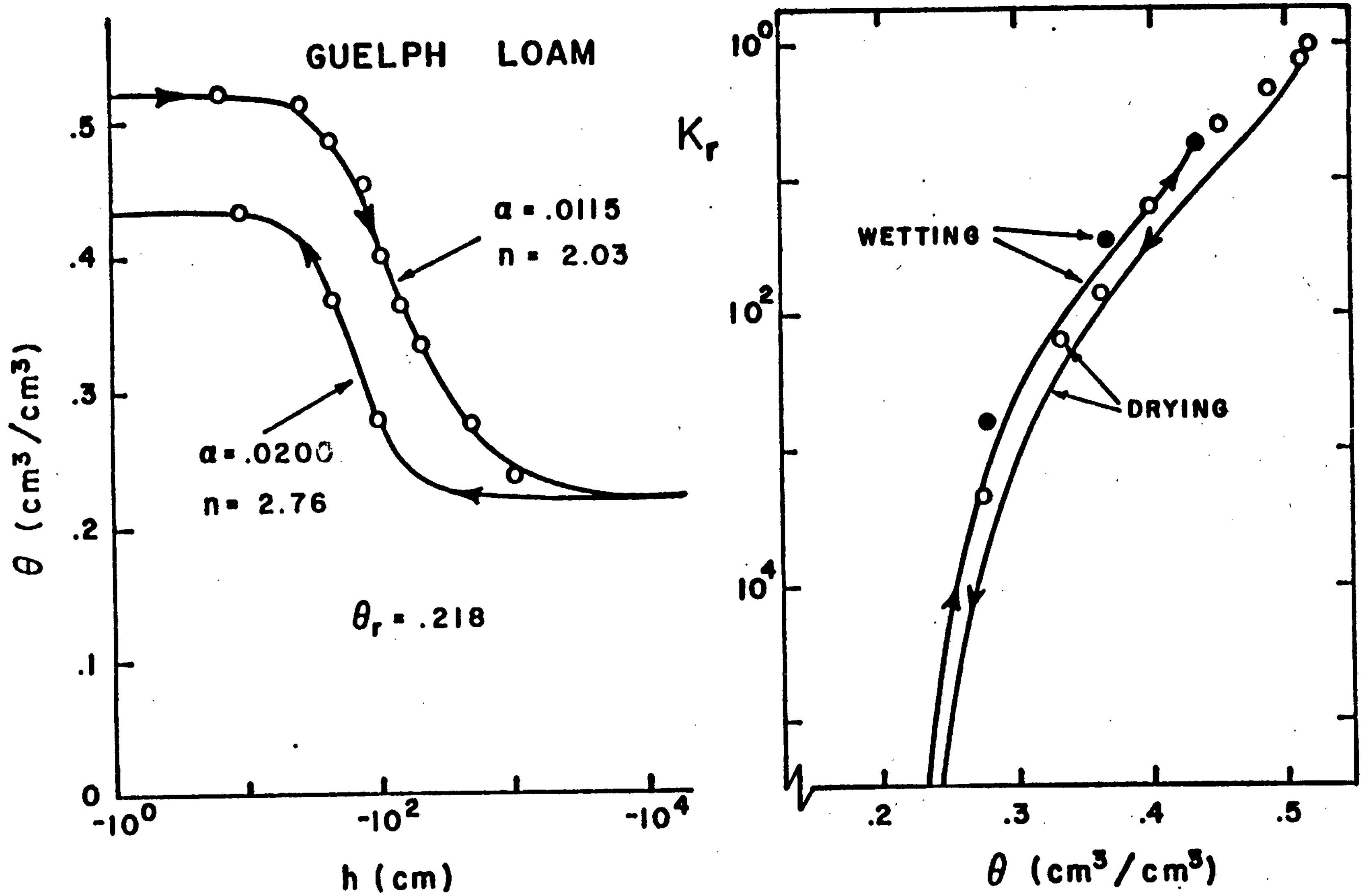

Fig. 14. Observed (circles) and calculated curves (solid lines) of the soil hydraulic properties of Guelph Loam. The drying and wetting branches of the relative hydraulic conductivity curve were predicted from knowledge of the curve-fitted branches of the soil moisture retention curve.

Information Only

retention curves are present, Eq. (8) also shows that different retention curves may generate the same conductivity curve as long as θ_r and m (and hence n) remain the same (i.e. α may be different).

Information Only

REFERENCES

Abramowitz, M., and I.A. Stegun. 1970. Handbook of mathematical functions. Dover Publ., New York. 1046 pp.

Ahuja, L.R., and D. Swartzendruber. 1972. An improved form of the soil-water diffusivity function. Soil Sci. Soc. Am. Proc. 36(1):9-14.

Amerman, C.R. 1976. Waterflow in soils: a generalized steady-state, two-dimensional porous media flow model. U.S. Dept. of Agr., ARS-NC-30. 62 pp.

Bresler, E. 1975. Two-dimensional transport of solutes during nonsteady infiltration from a trickle source. Soil Sci. Soc. Am. Proc. 39(4):604-613.

Brooks, R.H., and A.T. Corey. 1964. Hydraulic properties of porous media. Hydrology Paper No. 3, Civil Engineering Dept., Colorado State University, Fort Collins, Colorado.

Brooks, R.H., and A.T. Corey. 1966. Properties of porous media affecting fluid flow. J. Irrig. Drain. Div., Am. Soc. Civil Eng. 92(IR2): 61-88.

Bruce, R.R. 1972. Hydraulic conductivity evaluation of the soil profile from soil water retention relations. Soil Sci. Soc. Am. Proc. 36(4):555-561.

Burdine, N.T. 1953. Relative permeability calculations from pore-size distribution data. Petr. Trans., Am. Inst. Mining Metall. Eng. 198:71-77.

Daniel, C., and F.S. Wood. 1973. Fitting equations to data. Wiley-Interscience, New York. 350 pp.

Endelman, F.J., G.E.P. Box, J.R. Boyle, R.R. Hughes, D.R. Keeney, M.L. Northup, and P.G. Saffigna. 1974. The mathematical modeling of soil-water-nitrogen phenomena. Oak Ridge National Laboratory. EDFB-IBP-74-8. 66 pp.

Elrick, D.E., and D.H. Bowman. 1964. Note on an improved apparatus for soil moisture flow measurements. Soil Sci. Soc. Am. Proc. 28(3): 450-453.

Green, R.E., and J.C. Corey. 1971. Calculation of hydraulic conductivity: a further evaluation of some predictive methods. Soil Sci. Soc. Am. Proc. 35(1):3-8.

Haverkamp, R., M. Vauclin, J. Touma, P.J. Wierenga, and G. Vachaud. 1977. A comparison of numerical simulation models for one-dimensional infiltration. Soil Sci. Soc. Am. J. 41(2):285-294.

Information Only

Jackson, R.D. 1972. On the calculation of hydraulic conductivity. Soil Sci. Soc. Am. Proc. 36(2):380-382.

Jackson, R.D., R.J. Reginato, and C.H.M. van Bavel. 1965. Comparison of measured and calculated hydraulic conductivities of unsaturated soils. Water Resour. Res. 1:375-380.

Jeppson, R.W. 1974. Axisymmetric infiltration in soils, I. Numerical techniques for solution. J. Hydrol. 23:111-130.

Marquardt, D.W. 1963. An algorithm for least-squares estimation of non-linear parameters. J. Soc. Ind. Appl. Math. 11:431-441.

Meeter, D.A. 1964. Non-linear least-squares (Gaushaus). Univ. Wisc. Computing Center. Program revised 1966.

Millington, R.J., and J.P. Quirk. 1961. Permeability of porous solids. Trans. Faraday Soc. 57:1200-1206.

Mualem, Y. 1976a. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 12(3):513-522.

Mualem, Y. 1976b. A catalogue of the hydraulic properties of unsaturated soils. Research Project No. 442, Technion, Israel Institute of Technology, Haifa, Israel. 100 pp.

Rawitz, E. 1965. The influence of a number of environmental factors on the availability of soil moisture to plants (in Hebrew). Ph.D. thesis. Hebrew Univ., Rehovot, Israel.

Reeves, M., and J.O. Duguid. 1975. Water movement through saturated-undersaturated porous media: A finite-element galerkin model. Oak Ridge National Laboratory, ORNL-4927. 232 pp.

Reisenauer, A.E. 1963. Methods for solving problems of multidimensional partially saturated steady flow in soils. J. Geoph. Res. 68(20): 5725-5733.

Segol, G. 1976. A three-dimensional galerkin finite element model for the analysis of contaminant transport in variably saturated porous media. User's guide. Dept. of Earth Sciences, Univ. of Waterloo, Canada. 172 pp.

Information Only

APPENDIX A

SOHYP:

A COMPUTER MODEL FOR CALCULATING

THE SOIL HYDRAULIC PROPERTIES

FROM SOIL MOISTURE RETENTION DATA.

Information Only

This Appendix gives a brief description and listing of SOHYP, a computer program for calculation of the soil hydraulic properties from observed soil moisture retention data. The program does this by means of a non-linear least-squares fit of the following equation to the observed data [see also Eq. (28) in the text]

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{[1+(\alpha h)^n]^m} \tag{A1}$$

where for the Mualem theory,

$$m=1-1/n, \tag{A2}$$

and for the Burdine theory

$$m=1-2/n. \tag{A3}$$

The most significant variables in the program are defined in Table A1. Table A2 gives detailed instructions for set-up of the data cards, while Table A3 shows a list of the input data of example problem 3 (Silt Loam G.E.3), described in the main body of this report. The computer output for this example is given in Table A4, while the actual listing of the program is given in Table A5.

The computer program provides for three options, controlled by the variable MODE. If MODE equals one, the program optimizes the three parameters θ_r, α, and n by means of a least-squares fit of equations (A1) and (A2) to the observed data. The soil hydraulic properties are then calculated in accordance with the Mualem theory. If MODE equals two, the

Information Only

program only calculates best-fit values of α and n, and assumes that θ_r is known beforehand. The value of θ_r is now given as an input variable (see Table A2). Values of α and n are still calculated by means of Eq. (A1) and (A2) (i.e. the Mualem theory still applies). If MODE equals three, the computer model again calculates best-fit values of the three parameters (θ_r, α, and n), but it is now assumed that the Burdine theory applies. Hence Eq. (A1) and (A3) are now used in the program. In each case the computer program provides for a table of the hydraulic properties of the soil (see Table A4), consistent with the value of MODE selected.

Information Only

Table A1. List of the most significant variables in SOHYP.

VARIABLE	*DEFINITION*
AK	Hydraulic conductivity (K).
ALPHA	Coefficient α in Eq. (A1).
B(I)	Array containing initial estimates of coefficients.
BI(I)	Array of coefficient names.
DIFFUS	Soil moisture diffusivity (D).
MIT	Maximum number of iterations.
MODE	Designates model type to be used in program: =1: Three-parameter fit (θ_r, α, and n)(Mualem theory) =2: Two-parameter fit (α, n)(Mualem theory) =3: Three-parameter fit (θ_r, α, n)(Burdine theory).
MODEL	Subroutine to calculate soil moisture content (θ) from pressure head (Eq. A1).
NC	Number of cases considered.
NDATA	Input data code: =0: New data are read in =1: Data from previous case are used.
NIT	Iteration number during program execution.
NOB	Number of observed data points (must not exceed 40).
RK	Relative hydraulic conductivity (K_r).
RM	Equals 1-1/n for Mualem theory, 1-2/n for Burdine theory.
RN	Coefficient n in Eq. (A1).
RWC	Dimensionless moisture content (Θ).
SATK	Hydraulic conductivity at saturation (K_s).
SSQ, SUMB	Residual sum of squares.

Information Only

TABLE A1 (CONTINUED):

VARIABLE	DEFINITION
STOPCR	Stop Criterion. Iteration process stops when the relative change in each coefficient becomes less than STOPCR.
TITLE(I)	Array containing information of title cards.
WC	Volumetric moisture content (θ).
WCR	Residual moisture content (θ_r).
WCS	Saturated moisture content (θ_s).
X(I)	Array of observed pressure heads (values are assumed to be positive).
Y(I)	Array of observed moisture contents.

Information Only

Table A2.

CARD	COLUMNS	FORMAT	VARIABLE	COMMENT
1	1-5	I5	NC	Number of cases considered.
				The following cards are repeated NC times. However, skip cards 6, etc., if NDATA = 1 on third data card.
2	1-80	20(A4)	TITLE	
3	1-5	I5	MODE	Defines model number (1, 2, or 3).
	6-10	I5	NP	Number of coefficients (2 or 3).
	11-15	I5	NOB	Number of observations.
	16-20	I5	NDATA	Data input code.
	21-30	F10.0	WCR	Residual moisture content. This information is only necessary when MODE = 2.
	31-40	F10.0	WCS	Saturated moisture content.
	41-50	F10.0	SATK	Saturated hydraulic conductivity.
4	1-10	F10.0	B(1)	Initial value of θ_r if NP = 3; Initial value of α if NP = 2.
	11-20	F10.0	B(2)	Initial value of α if NP = 3; Initial value of n if NP = 2.
	21-30	F10.0	B(3)	Initial value of n if NP = 3.
5	1-6	A4,A2	BI(1)	Coefficient name of B(1).
	11-16	A4,A2	BI(2)	Coefficient name of B(2).
	21-26	A4,A2	BI(3)	Coefficient name of B(3) (only if NP = 3).
6,etc.	1-10	F10.0	X(I)	Value of observed pressure head (assumed to be positive).
	11-20	F10.0	Y(I)	Value of observed moisture content.

Information Only

Table A3. Input data for example 3 (Silt Loam G.E.3).

```
                      1         2         3         4         5
Column:      12345678901234567890123456789012345678901234567890

Card

   1            1
   2                 SILT LOAM G.E.3
   3            1    3   13    0      0.18     0.396      4.96
   4           0.180    0.002     2.3
   5          WCR      ALPHA      N
   6            10.0   0.396
   7            20.0   0.394
   8            43.0   0.390
   9            60.0   0.3855
  10            80.0   0.379
  11           111.0   0.370
  12           190.0   0.340
  13           285.0   0.300
  14           400.0   0.260
  15           600.0   0.220
  16           800.0   0.200
  17           900.0   0.194
  18          1000.0   0.190
```


Information Only

Table A4. Output for example 3 (Silt Loam G.E.3).

Information Only

```
*****************************************************************
*                                                               *
*        NCN-LINEAR LEAST SQUARES ANALYSIS                      *
*                                                               *
*        SILT LOAM G.E.3                                        *
*                                                               *
*****************************************************************
```

```
INPUT PARAMETERS
================
MODEL NUMBER.........................................   1
NUMBER OF COEFFICIENTS...............................   3
NUMBER OF OBSERVATIONS...............................  13
RESIDUAL MOISTURE CONTENT (FCR MODEL 2).......     0.1800
SATURATED MOISTURE CCNTENT....................     0.3960
SATURATED HYDRAULIC CONDUCTIVITY..............     4.9600
```

OBSERVED DATA
=============

OBS. NO.	PRESSURE HEAD	MOISTURE CONTENT
1	10.00	0.3960
2	20.00	0.3940
3	43.00	0.3900
4	60.00	0.3855
5	80.00	0.3790
6	111.00	0.3700
7	190.00	0.3400
8	285.00	0.3000
9	400.00	0.2600
10	600.00	0.2200
11	800.00	0.2000
12	900.00	0.1940
13	1000.00	0.1900

Information Only

ITERATION NO	WCR	ALPHA	N	SSQ	MODEL
0	0.1800	0.002000	2.5000	0.0344679	1
1	0.1377	0.003084	2.2168	0.0038162	1
2	0.1329	0.004015	2.0421	0.0003537	1
3	0.1324	0.004237	2.0656	0.0000668	1
4	0.1314	0.004232	2.0597	0.0000665	1
5	0.1313	0.004233	2.0594	0.0000665	1

CORRELATION MATRIX
==================

	1	2	3
1	1.0000		
2	0.3543	1.0000	
3	0.9466	0.0831	1.0000

NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS
==

VARIABLE	VALUE	S.E.COEFF.	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
WCR	0.13132	0.0096	13.67	0.1099	0.1527
ALPHA	0.00423	0.0001	44.40	0.0040	0.0044
N	2.05945	0.0805	25.59	1.8802	2.2387

--------ORDERED BY COMPUTER INPUT--------

NO	PRESSURE	MOISTURE CONTENT OBS	FITTED	RESI-DUAL
1	10.00	0.3960	0.3958	0.0002
2	20.00	0.3940	0.3952	-0.0012
3	43.00	0.3900	0.3920	-0.0020
4	60.00	0.3855	0.3883	-0.0028
5	80.00	0.3790	0.3825	-0.0035
6	111.00	0.3700	0.3712	-0.0012
7	190.00	0.3400	0.3366	0.0034
8	285.00	0.3000	0.2975	0.0025
9	400.00	0.2600	0.2618	-0.0018
10	600.00	0.2200	0.2232	-0.0032
11	800.00	0.2000	0.2012	-0.0012
12	900.00	0.1940	0.1935	0.0005
13	1000.00	0.1900	0.1873	0.0027

----------ORDERED BY RESIDUALS----------

NO	PRESSURE	MOISTURE CONTENT OBS	FITTED	RESI-DUAL
7	190.00	0.3400	0.3366	0.0034
13	1000.00	0.1900	0.1873	0.0027
8	285.00	0.3000	0.2975	0.0025
12	900.00	0.1940	0.1935	0.0005
1	10.00	0.3960	0.3958	0.0002
11	800.00	0.2000	0.2012	-0.0012
2	20.00	0.3940	0.3952	-0.0012
6	111.00	0.3700	0.3712	-0.0012
9	400.00	0.2600	0.2618	-0.0018
3	43.00	0.3900	0.3920	-0.0020
4	60.00	0.3855	0.3863	-0.0028
10	600.00	0.2200	0.2232	-0.0032
5	80.00	0.3790	0.3825	-0.0035

Information Only

PRESSURE	LOG P	WC	REL K	LOG RK	ABS K	LOG KA	DIFFUS	LOG D
0.0		0.3960	0.100E 01		0.496E 01			
0.141E 01	0.150	0.3960	0.991E 00	-0.004	0.492E 01	0.692	0.939E 06	5.973
0.168E 01	0.225	0.3960	0.989E 00	-0.005	0.491E 01	0.691	0.781E 06	5.893
0.200E 01	0.300	0.3960	0.987E 00	-0.006	0.490E 01	0.690	0.649E 06	5.812
0.237E 01	0.375	0.3960	0.985E 00	-0.007	0.488E 01	0.689	0.539E 06	5.732
0.282E 01	0.450	0.3960	0.982E 00	-0.008	0.487E 01	0.687	0.448E 06	5.651
0.335E 01	0.525	0.3960	0.978E 00	-0.010	0.485E 01	0.686	0.371E 06	5.570
0.398E 01	0.600	0.3960	0.974E 00	-0.012	0.483E 01	0.684	0.308E 06	5.488
0.473E 01	0.675	0.3960	0.968E 00	-0.014	0.480E 01	0.682	0.255E 06	5.407
0.562E 01	0.750	0.3959	0.962E 00	-0.017	0.477E 01	0.679	0.211E 06	5.325
0.668E 01	0.825	0.3959	0.955E 00	-0.020	0.473E 01	0.675	0.174E 06	5.242
0.794E 01	0.900	0.3959	0.946E 00	-0.024	0.469E 01	0.671	0.144E 06	5.158
0.944E 01	0.975	0.3958	0.935E 00	-0.029	0.464E 01	0.666	0.119E 06	5.074
0.112E 02	1.050	0.3957	0.922E 00	-0.035	0.457E 01	0.660	0.975E 05	4.989
0.133E 02	1.125	0.3956	0.907E 00	-0.043	0.450E 01	0.653	0.800E 05	4.903
0.158E 02	1.200	0.3955	0.888E 00	-0.051	0.441E 01	0.644	0.654E 05	4.815
0.188E 02	1.275	0.3953	0.867E 00	-0.062	0.430E 01	0.633	0.532E 05	4.726
0.224E 02	1.350	0.3949	0.841E 00	-0.075	0.417E 01	0.620	0.432E 05	4.635
0.266E 02	1.425	0.3945	0.811E 00	-0.091	0.402E 01	0.604	0.348E 05	4.542
0.316E 02	1.500	0.3939	0.775E 00	-0.111	0.384E 01	0.585	0.279E 05	4.446
0.376E 02	1.575	0.3930	0.734E 00	-0.134	0.364E 01	0.561	0.222E 05	4.347
0.447E 02	1.650	0.3917	0.686E 00	-0.164	0.340E 01	0.532	0.176E 05	4.245
0.531E 02	1.725	0.3899	0.631E 00	-0.200	0.313E 01	0.496	0.137E 05	4.138
0.631E 02	1.800	0.3874	0.570E 00	-0.244	0.283E 01	0.451	0.106E 05	4.026
0.750E 02	1.875	0.3840	0.502E 00	-0.299	0.249E 01	0.396	0.811E 04	3.909
0.891E 02	1.950	0.3794	0.430E 00	-0.367	0.213E 01	0.329	0.611E 04	3.786
0.106E 03	2.025	0.3732	0.355E 00	-0.450	0.176E 01	0.246	0.453E 04	3.656
0.126E 03	2.100	0.3650	0.281E 00	-0.551	0.139E 01	0.144	0.330E 04	3.518
0.150E 03	2.175	0.3547	0.212E 00	-0.675	0.105E 01	0.021	0.236E 04	3.373
0.178E 03	2.250	0.3421	0.151E 00	-0.822	0.747E 00	-0.127	0.166E 04	3.221
0.211E 03	2.325	0.3272	0.101E 00	-0.996	0.500E 00	-0.301	0.115E 04	3.061
0.251E 03	2.400	0.3105	0.634E-01	-1.198	0.315E 00	-0.502	0.783E 03	2.894
0.299E 03	2.475	0.2926	0.375E-01	-1.426	0.186E 00	-0.730	0.526E 03	2.721
0.355E 03	2.550	0.2743	0.210E-01	-1.678	0.104E 00	-0.983	0.349E 03	2.543
0.422E 03	2.625	0.2563	0.112E-01	-1.953	0.553E-01	-1.257	0.230E 03	2.361
0.501E 03	2.700	0.2394	0.569E-02	-2.245	0.282E-01	-1.549	0.150E 03	2.176
0.596E 03	2.775	0.2238	0.281E-02	-2.551	0.139E-01	-1.856	0.973E 02	1.988
0.708E 03	2.850	0.2100	0.135E-02	-2.869	0.670E-02	-2.174	0.629E 02	1.798
0.841E 03	2.925	0.1978	0.637E-03	-3.196	0.316E-02	-2.500	0.405E 02	1.608
0.100E 04	3.000	0.1873	0.296E-03	-3.529	0.147E-02	-2.833	0.260E 02	1.416

Information Only

0.119E 04	3.075	0.1783	0.136E-03	-3.866	0.676E-03	-3.170	0.167E 02	1.223
0.141E 04	3.150	0.1706	0.622E-04	-4.206	0.308E-03	-3.511	0.107E 02	1.030
0.168E 04	3.225	0.1642	0.282E-04	-4.549	0.140E-03	-3.854	0.687E 01	0.837
0.200E 04	3.300	0.1588	0.128E-04	-4.894	0.633E-04	-4.199	0.440E 01	0.643
0.237E 04	3.375	0.1542	0.576E-05	-5.240	0.286E-04	-4.544	0.282E 01	0.450
0.282E 04	3.450	0.1504	0.259E-05	-5.586	0.129E-04	-4.891	0.180E 01	0.256
0.335E 04	3.525	0.1472	0.117E-05	-5.934	0.578E-05	-5.238	0.115E 01	0.062
0.398E 04	3.600	0.1446	0.522E-06	-6.282	0.259E-05	-5.587	0.736E 00	-0.133
0.473E 04	3.675	0.1424	0.235E-06	-6.630	0.116E-05	-5.934	0.471E 00	-0.327
0.562E 04	3.750	0.1405	0.105E-06	-6.978	0.522E-06	-6.282	0.301E 00	-0.521
0.668E 04	3.825	0.1390	0.472E-07	-7.326	0.234E-06	-6.631	0.193E 00	-0.715
0.794E 04	3.900	0.1377	0.212E-07	-7.674	0.105E-06	-6.979	0.123E 00	-0.909
0.944E 04	3.975	0.1366	0.949E-08	-8.023	0.471E-07	-7.327	0.789E-01	-1.103
0.112E 05	4.050	0.1357	0.425E-08	-8.371	0.211E-07	-7.676	0.504E-01	-1.297
0.133E 05	4.125	0.1350	0.191E-08	-8.720	0.945E-08	-8.024	0.323E-01	-1.491
0.158E 05	4.200	0.1344	0.854E-09	-9.068	0.424E-08	-8.373	0.206E-01	-1.686
0.188E 05	4.275	0.1339	0.383E-09	-9.417	0.190E-08	-8.722	0.132E-01	-1.880
0.224E 05	4.350	0.1335	0.172E-09	-9.766	0.851E-09	-9.070	0.843E-02	-2.074
0.266E 05	4.425	0.1331	0.769E-10	-10.114	0.381E-09	-9.419	0.539E-02	-2.268
0.316E 05	4.500	0.1328	0.344E-10	-10.463	0.171E-09	-9.767	0.345E-02	-2.462
0.376E 05	4.575	0.1326	0.154E-10	-10.812	0.766E-10	-10.116	0.221E-02	-2.656
0.447E 05	4.650	0.1323	0.692E-11	-11.160	0.343E-10	-10.465	0.141E-02	-2.851
0.531E 05	4.725	0.1322	0.310E-11	-11.509	0.154E-10	-10.813	0.902E-03	-3.045
0.631E 05	4.800	0.1320	0.139E-11	-11.857	0.689E-11	-11.162	0.577E-03	-3.239
0.750E 05	4.875	0.1319	0.622E-12	-12.206	0.309E-11	-11.511	0.369E-03	-3.433
0.891E 05	4.950	0.1318	0.279E-12	-12.555	0.138E-11	-11.859	0.236E-03	-3.627
0.106E 06	5.025	0.1317	0.125E-12	-12.903	0.620E-12	-12.208	0.151E-03	-3.822
0.126E 06	5.100	0.1317	0.560E-13	-13.252	0.278E-12	-12.556	0.964E-04	-4.016
0.150E 06	5.175	0.1316	0.251E-13	-13.601	0.124E-12	-12.905	0.617E-04	-4.210
0.178E 06	5.250	0.1316	0.112E-13	-13.949	0.557E-13	-13.254	0.394E-04	-4.404
0.211E 06	5.325	0.1315	0.504E-14	-14.298	0.250E-13	-13.602	0.252E-04	-4.598
0.251E 06	5.400	0.1315	0.226E-14	-14.647	0.112E-13	-13.951	0.161E-04	-4.792
0.298E 06	5.475	0.1315	0.101E-14	-14.995	0.502E-14	-14.300	0.103E-04	-4.987
0.355E 06	5.550	0.1314	0.453E-15	-15.344	0.225E-14	-14.648	0.659E-05	-5.181
0.422E 06	5.625	0.1314	0.203E-15	-15.692	0.101E-14	-14.997	0.422E-05	-5.375
0.501E 06	5.700	0.1314	0.910E-16	-16.041	0.451E-15	-15.346	0.270E-05	-5.569

Information Only

Table A5. Fortran listing of SOHYP.

Information Only

```
C
C
C     ************************************************************************
C     *                                                                      *
C     *      NCN-LINEAR LEAST-SQUARES ANALYSIS OF                   SOHYP   *
C     *      SOIL HYDRAULIC PROPERTIES                           APRIL 1980  *
C     *                                                                      *
C     ************************************************************************
C
C
      DIMENSION X(40),Y(40),R(40),F(40),DELZ(40,4),LSORT(40),B(3),BI(6),
     1E(3),P(3),PHI(3),Q(3),TB(3),A(3,3),D(3,3),TITLE(20),TH(3)
      DATA STOPCR/.0010/,MIT/20/
C
C     ----- REAC NUMBER OF CASES CONSIDERED -----
      READ(5,1000) NC
      DC 144 IC=1,NC
      READ(5,10C2) TITLE
      WRITE(6,1004) TITLE
C
C     ----- READ INPUT PARAMETERS -----
      READ(5,1000) MCDE,NP,NOB,NDATA,WCR,WCS,SATK
      WRITE(6,1005) MCDE,NP,NOB,WCR,WCS,SATK
C
C     ----- READ INITIAL ESTIMATES -----
      READ(5,10C6) (B(I),I=1,NP)
C
C     ----- REAC COEFFICIENTS NAMES -----
      NBI=2*NP
      READ(5,1007) (BI(I),I=1,NBI)
C
C     ----- REAC AND WRITE EXPERIMENTAL DATA -----
      WRITE(6,1008)
      IF(NCATA.GT.0) GO TO 8
      DO 4 I=1,NOB
    4 REAC(5,10C6) X(I),Y(I)
    8 DC 10 I=1,NOB
   10 WRITE(6,1011) I,X(I),Y(I)
C
C     ----------
      DO 12 I=1,NP
   12 TH(I)=B(I)
      IF((NP-2)*(NP-3)) 14,16,14
   14 WRITE(6,1016)
      GO TC 142
   16 GA=0.02
      CALL MODEL(TH,F,NOB,X,WCS,MODE,NP,WCR)
      SSC=0.
      DO 32 I=1,NOB
      R(I)=Y(I)-F(I)
   32 SSC=SSQ+R(I)*R(I)
      NIT=0
      WRITE(6,1030)
      IF(MODE.EQ.2) WRITE(6,1026) NIT,WCR,B(1),B(2),SSQ,MODE
```


Information Only

```
      IF(MODE.NE.2) WRITE(6,1026) NIT,B(1),B(2),B(3),SSQ,MODE
C
C     ----- BEGIN OF ITERATION -----
   34 NIT=NIT+1
      GA=0.1*GA
      DO 38 J=1,NP
      TEMP=TH(J)
      TH(J)=1.01*TH(J)
      Q(J)=0
      CALL MODEL(TH,DELZ(1,J),NOB,X,WCS,MODE,NP,WCR)
      DO 36 I=1,NOB
      DELZ(I,J)=DELZ(I,J)-F(I)
   36 Q(J)=Q(J)+DELZ(I,J)*R(I)
      Q(J)=100.*C(J)/TH(J)
C
C     ----- STEEPEST CESCENT -----
   38 TH(J)=TEMP
      DO 44 I=1,NP
      DO 42 J=1,I
      SUM=0
      DO 40 K=1,NOB
   40 SUM=SUM+DELZ(K,I)*DELZ(K,J)
      D(I,J)=10000.*SUM/(TH(I)*TH(J))
   42 D(J,I)=D(I,J)
C
C     ----- D = MOMENT MATRIX -----
   44 E(I)=SQRT(D(I,I))
   50 DO 52 I=1,NP
      DO 52 J=1,NP
   52 A(I,J)=D(I,J)/(E(I)*E(J))
C
C     ----- A IS THE SCALED MOMENT MATRIX -----
      DO 54 I=1,NP
      P(I)=Q(I)/E(I)
      PHI(I)=P(I)
   54 A(I,I)=A(I,I)+GA
      CALL MATINV(A,NP,P)
C
C     ----- P/E IS THE CORRECTION VECTOR -----
      STEP=1.0
   56 DO 58 I=1,NP
   58 TB(I)=P(I)*STEP/E(I)+TH(I)
      DO 62 I=1,NP
      IF(TH(I)*TB(I))66,66,62
   62 CONTINUE
      SUMB=0.0
      CALL MODEL(TB,F,NOB,X,WCS,MODE,NP,WCR)
      DO 64 I=1,NCB
      R(I)=Y(I)-F(I)
   64 SUMB=SUMB+R(I)*R(I)
   66 SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      DO 68 I=1,NP
```


Information Only

```
      SUM1=SUM1+P(I)*PHI(I)
      SUM2=SUM2+P(I)*P(I)
   68 SUM3=SUM3+PHI(I)*PHI(I)
      ANGLE=57.29578*ARCOS(SUM1/SQRT(SUM2*SUM3))
C
C     ----------
      DO 72 I=1,NP
      IF(TH(I)*TB(I))74,74,72
   72 CONTINUE
      IF(SUMB/SSQ-1.0)80,80,74
   74 IF(ANGLE-30.0)76,76,78
   76 STEP=STEP/2.0
      GO TO 56
   78 GA=10.*GA
      GO TO 50
C
C     ----- PRINT COEFFICIENTS AFTER EACH ITERATION -----
   80 CONTINUE
      DO 82 I=1,NP
   82 TH(I)=TB(I)
      IF(MODE.EQ.2) WRITE(6,1026) NIT,WCR,TH(1),TH(2),SUMB,MODE
      IF(MODE.NE.2) WRITE(6,1026) NIT,TH(1),TH(2),TH(3),SUMB,MODE
      IF(MODE.EQ.2) GO TO 90
      IF(TH(1).GT.0.005) GO TO 90
      WRITE(6,1028)
      GO TO 144
   90 DO 92 I=1,NP
      IF(ABS(P(I)*STEP/E(I))/(1.0E-20+ABS(TH(I)))-STOPCR) 92,92,94
   92 CONTINUE
      GO TO 96
   94 SSQ=SUMB
      IF(NIT.LE.MIT) GO TO 34
C
C     ----- END OF ITERATION LOOP -----
   96 CONTINUE
      CALL MATINV(D,NP,P)
C
C     ----- WRITE CORRELATION MATRIX -----
      DO 98 I=1,NP
   98 E(I)=SQRT(D(I,I))
      WRITE(6,1044) (I,I=1,NP)
      DO 102 I=1,NP
      DO 100 J=1,I
  100 A(J,I)=D(J,I)/(E(I)*E(J))
  102 WRITE(6,1048) I,(A(J,I),J=1,I)
C
C     ----- CALCULATE 95% CONFIDENCE INTERVAL -----
      Z=1./FLOAT(NOB-NP)
      SDEV=SQRT(Z*SUMB)
      WRITE(6,1052)
      TVAR=1.96+Z*(2.3779+Z*(2.7135+Z*(3.187936+2.466666*Z**2)))
      DO 108 I=1,NP
      SECOEF= E(I)*SDEV
      TVALUE= TH(I)/SECOEF
```


Information Only

```
      TSEC=TVAR*SECOEF
      TMCOE=TH(I)-TSEC
      TPCOE=TH(I)+TSEC
      K=2*I
      J=K-1
  108 WRITE(6,1058) BI(J),BI(K),TH(I),SECOEF,TVALUE,TMCOE,TPCOE
C
C     ----- PREPARE FINAL OUTPUT -----
      LSORT(1)=1
      DO 116 J=2,NOB
      TEMP=R(J)
      K=J-1
      DO 111 L=1,K
      LL=LSORT(L)
      IF(TEMP-R(LL)) 112,112,111
  111 CONTINUE
      LSORT(J)=J
      GO TO 116
  112 KK=J
  113 KK=KK-1
      LSORT(KK+1)=LSORT(KK)
      IF(KK-L) 115,115,113
  115 LSORT(L)=J
  116 CONTINUE
      WRITE(6,1066)
      DO 118 I=1,NOB
      J=LSORT(NOB+1-I)
  118 WRITE(6,1068) I,X(I),Y(I),F(I),R(I),J,X(J),Y(J),F(J),R(J)
C
C     ----- WRITE SOIL HYDRAULIC PROPERTIES -----
      WRITE(6,1069)
      PRESS=1.18850
      RN1=0.0
      RKLN=1.0
      WRITE(6,1072) RN1,WCS,RKLN,SATK
      DO 140 I=1,75
      IF(RKLN.LT.(-16.)) GO TO 142
      PRESS=1.18850*PRESS
      IF(MODE-2) 120,122,120
  120 WCR=TH(1)
      ALPHA=TH(2)
      RN=TH(3)
      GO TO 124
  122 ALPHA=TH(1)
      RN=TH(2)
  124 RM=1.-1./RN
      IF(MODE.EQ.3) RM=1.-2./RN
      RN1=RM*RN
      RWC=1./(1.+(ALPHA*PRESS)**RN)**RM
      WC=WCR+(WCS-WCR)*RWC
      TERM=1.-RWC*(ALPHA*PRESS)**RN1
      IF((TERM.LT.5.E-05).OR.(RWC.LT.0.06)) TERM = RM*RWC**(1./RM)
      IF(MODE.EQ.3) RK=RWC*RWC*TERM
      IF(MODE.NE.3) RK=SQRT(RWC)*TERM*TERM
```


Information Only

```
      TERM=ALPHA*RN1*(WCS-WCR)*RWC*RWC**(1./RM)*(ALPHA*PRESS)**(RN-1.)
      AK=SATK*RK
      DIFFUS=AK/TERM
      PRLN=ALOG10(PRESS)
      AKLN=ALOG10(AK)
      RKLN=ALOG10(RK)
      DIFLN=ALOG10(DIFFUS)
  140 WRITE(6,1070) PRESS,PRLN,WC,RK,RKLN,AK,AKLN,DIFFUS,DIFLN
  142 CONTINUE
  144 CONTINUE
C
C     ----- END OF PROBLEM -----
 1000 FORMAT(4I5,5F10.0)
 1002 FORMAT(20A4)
 1004 FORMAT(1H1,10X,82(1H*)/11X,1H*,80X,1H*/11X,1H*, 9X,'NON-LINEAR LEA
     1ST SQUARES ANALYSIS',38X,1H*/11X,1H*,80X,1H*/11X,1H*,20A4,1H*/11X,
     21H*,80X,1H*/11X,82(1H*))
 1005 FORMAT(//11X,'INPUT PARAMETERS'/11X,16(1H=)/
     211X,'MODEL NUMBER...................................',I3/
     311X,'NUMBER OF COEFFICIENTS.........................',I3/
     411X,'NUMBER OF OBSERVATIONS.........................',I3/
     511X,'RESIDUAL MOISTURE CONTENT (FOR MODEL 2)........',F10.4/
     611X,'SATURATED MOISTURE CONTENT.....................',F10.4/
     711X,'SATURATED HYDRAULIC CONDUCTIVITY...............',F10.4)
 1006 FORMAT(4F10.0)
 1007 FORMAT(4(A4,A2,4X))
 1008 FORMAT(//11X,'OBSERVED DATA',/11X,13(1H=)/11X,'OBS. NO.',4X,'PRESS
     1URE HEAD',2X,'MOISTURE CONTENT')
 1011 FORMAT(11X,I5,5X,F12.2,4X,F12.4)
 1016 FORMAT(//5X,10(1H*),' ERROR: INCORRECT NUMBER OF COEFFICIENTS')
 1026 FORMAT(15X,I2,10X,F8.4,3X,F10.6,2X,F10.4,5X,F12.7,4X,I4)
 1028 FORMAT(//11X,'WCR IS LESS THAN 0.005, USE TWO-PARAMETER MODEL WITH
     1 WCR = 0.0')
 1030 FORMAT(1H1,10X,'ITERATION NO',8X,'WCR',8X,'ALPHA',10X,'N',13X,'SSQ
     1',8X,'MODEL')
 1044 FORMAT(//11X,'CORRELATION MATRIX'/11X,18(1H=)/14X,10(4X,I2,5X))
 1048 FORMAT(11X,I3,10(2X,F7.4,2X))
 1052 FORMAT(//11X,'NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS'/
     111X,48(1H=)/64X,'95% CONFIDENCE LIMITS'/11X,'VARIABLE',8X,'VALUE',
     27X,'S.E.COEFF.',3X,'T-VALUE',6X,'LOWER',10X,'UPPER')
 1058 FORMAT(13X,A4,A2,4X,F10.5,5X,F9.4,5X,F6.2,4X,F9.4,5X,F9.4)
 1066 FORMAT(//10X,8(1H-),'ORDERED BY COMPUTER INPUT', 8(1H-), 7X,10(1H-
     1),'ORDERED BY RESIDUALS',10(1H-)/26X,'MOISTURE CONTENT',3X,'RESI-'
     1,24X,'MOISTURE CONTENT',3X,'RESI-'/10X,'NO',3X,'PRESSURE',5X,'OBS'
     2,4X,'FITTED',4X,'DUAL', 9X,'NO',3X,'PRESSURE',5X,'OBS',4X,'FITTED'
     3,4X,'DUAL')
 1068 FORMAT(10X,I2,F10.2,1X,3F9.4,8X,I2,F10.2,1X,3F9.4)
 1069 FORMAT(1H1,10X,'PRESSURE',4X,'LOG P',6X,'WC',7X,'REL K',5X,'LOG RK
     1',6X,'ABS K',4X,'LOG KA',5X,'DIFFUS',5X,'LOG D')
 1070 FORMAT(10X,E10.3,F8.3,F10.4,3(E13.3,F8.3))
 1072 FORMAT(10X,E10.3,8X,F10.4,E13.3,8X,E13.3)
      STOP
      END
```


Information Only

MATINV

```
      SUBROUTINE MATINV(A,NP,B)
      DIMENSION A(3,3),B(3),INDEX(3,2)
      DO 2 J=1,4
    2 INDEX(J,1)=0
      I=0
    4 AMAX=-1.0
      DO 10 J=1,NP
      IF(INDEX(J,1)) 10,6,10
    6 DO 10 K=1,NP
      IF(INDEX(K,1)) 10,8,10
    8 P=ABS(A(J,K))
      IF(P.LE.AMAX) GO TO 10
      IR=J
      IC=K
      AMAX=P
   10 CONTINUE
      IF(AMAX) 30,30,14
   14 INDEX(IC,1)=IR
      IF(IR.EQ.IC) GO TO 18
      DO 16 L=1,NP
      P=A(IR,L)
      A(IR,L)=A(IC,L)
   16 A(IC,L)=P
      P=B(IR)
      B(IR)=B(IC)
      B(IC)=P
      I=I+1
      INDEX(I,2)=IC
   18 P=1./A(IC,IC)
      A(IC,IC)=1.0
      DO 20 L=1,NP
   20 A(IC,L)=A(IC,L)*P
      B(IC)=B(IC)*P
      DO 24 K=1,NP
      IF(K.EQ.IC) GO TO 24
      P=A(K,IC)
      A(K,IC)=0.0
      DO 22 L=1,NP
   22 A(K,L)=A(K,L)-A(IC,L)*P
      B(K)=B(K)-B(IC)*P
   24 CONTINUE
      GO TO 4
   26 IC=INDEX(I,2)
      IR=INDEX(IC,1)
      DO 28 K=1,NP
      P=A(K,IR)
      A(K,IR)=A(K,IC)
   28 A(K,IC)=P
      I=I-1
   30 IF(I) 26,32,26
   32 RETURN
      END
```


Information Only

MODEL

```
      SUBROUTINE MODEL(B,FY,NOB,X,WCS,MODE,NP,WCR)
      DIMENSION B(3),FY(40),X(40)
C
C     MODE=1 : MUALEM THEORY WITH THREE COEFFICIENTS
C     MODE=2 : MUALEM THEORY WITH TWO COEFFICIENTS
C     MODE=3 : BURDINE THEORY WITH THREE COEFFICIENTS
C
      IF(MODE-2) 10,20,30
   10 CONTINUE
      DO 12 J=1,NOB
   12 FY(J)=B(1)+(WCS-B(1))/(1.+(B(2)*X(J))**B(3))**(1.-1./B(3))
      RETURN
   20 CONTINUE
      DO 22 J=1,NOB
   22 FY(J)=WCR+(WCS-WCR)/(1.+(B(1)*X(J))**B(2))**(1.-1./B(2))
      RETURN
   30 CONTINUE
      DO 32 J=1,NOB
   32 FY(J)=B(1)+(WCS-B(1))/(1.+(B(2)*X(J))**B(3))**(1.-2./B(3))
      RETURN
      END
```


Information Only