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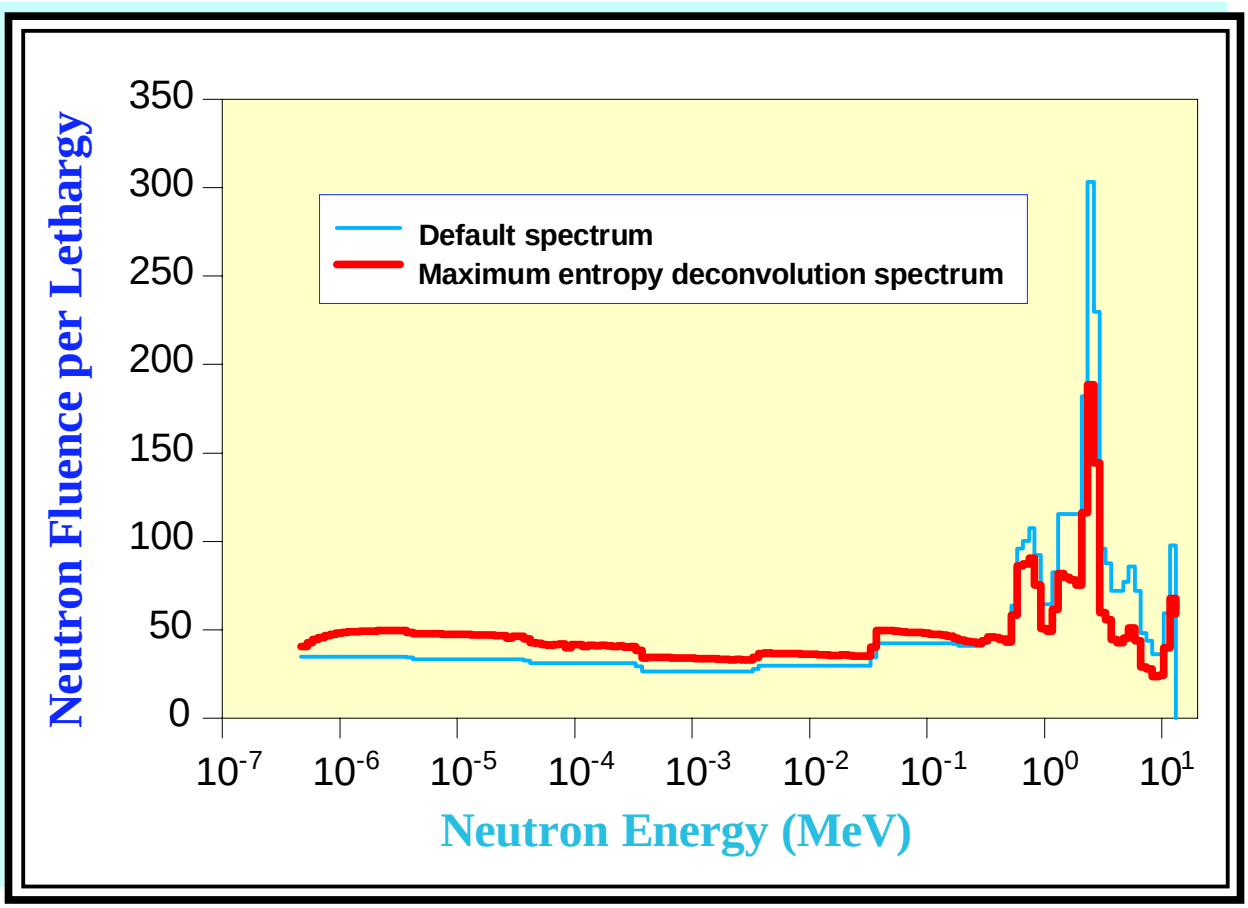
EML-595

MAXED, A Computer Code For The Deconvolution Of Multisphere Neutron Spectrometer Data Using The Maximum Entropy Method



Marcel Reginatto and Paul Goldhagen

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MAXED, A COMPUTER CODE FOR THE DECONVOLUTION OF MULTISPHERE NEUTRON SPECTROMETER DATA USING THE MAXIMUM ENTROPY METHOD

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ABSTRACT

The problem of analyzing data from a multisphere neutron spectrometer to infer the energy spectrum of the incident neutrons is discussed. The main features of the code MAXED, a computer program developed to apply the maximum entropy principle to the deconvolution (unfolding) of multisphere neutron spectrometer data, are described, and the use of the code is illustrated with an example. A user's guide for the code MAXED is included in an appendix. The code is available from the authors upon request.

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INTRODUCTION

The multisphere neutron spectrometer (also known as Bonner spheres after one of the authors of the original paper) was first described by Bramblett et al. (1960). Such a spectrometer is a set of detector assemblies made of spherical moderators of different sizes, each one with a counter sensitive to thermal neutrons placed at the center. Each detector is characterized by a response function, which is the efficiency of the detector as a function of incident neutron energy. The shape of the response function depends on the size of the moderator. As the size of the moderator is increased, the peak of the response function shifts to a higher neutron energy.

The number of counts, $N_{k,}$ measured by detector k is related to the detector's response function, $R_k(E)$, and the neutron spectrum, f(E), by an integral over the energy of the form

$$N_k + \epsilon_k = \int R_k(E) f(E) dE \qquad (1.1)$$

where ϵ_k is the (unknown) measurement error in detector k. Figure 1 shows a set of response functions for a multisphere neutron spectrometer that has been used by the Environmental Measurements Laboratory [for more details see Goldhagen et al. (1996)]. The 12 detectors are spherical 5.08 cm-diameter $^{10}BF_3$-filled stainless steel pulse ionization chambers, one of them is bare (unshielded), one is surrounded with a layer of cadmium, and the remaining 10 are surrounded with high-density polyethylene spheres with diameters ranging from 7.4 to 30 cm. To reduce the response to thermal neutrons and improve energy resolution, the moderator spheres are surrounded with cadmium shells and then aluminum shells for safe handling. The sharp dips in the response functions come from nuclear resonances in the cadmium and aluminum shells surrounding the spheres and, for spheres 11 and 12, in the carbon of the polyethylene moderators.

The problem of deriving f(E) from a finite number of measurements [i.e., finding f(E) that solves Equation (1.1) for all measurements] does not have a unique solution. This can be understood intuitively -- without further assumptions it is clearly not possible to determine a continuous function such as f(E) from a finite number of measurements. Non-uniqueness would not be a serious problem if for all practical purposes the measured data was sufficient to force all

solutions of Equation (1.1) to be very similar. But this is not the case for multisphere spectrometers, where the energy response functions are not sharply peaked and overlap over many orders of magnitude (see Figure 1).

The process of choosing a particular solution spectrum from the infinite number of spectra that fit the data is known as deconvolution (or unfolding) of the data. To make this choice, we need to find additional criteria to supplement the set of constraints imposed by the measurements. While many deconvolution algorithms use criteria that are purely mathematical, we believe that to choose a physically relevant spectrum it is crucial to use *a priori* information.

In this paper, we take the point of view that deconvolution of data is best viewed as a problem of inference. That is, to deconvolve our data we must take an initial estimate of the spectrum (which encodes the *a priori* information that is available *before* the measurements are made) and modify it using information from a set of new measurements. This new, improved spectrum becomes our best estimate. If the method of deconvolution used is sound and we have good *a priori* information available, we expect our final best estimate to be close to the true spectrum. Which method of deconvolution if any is the correct one for this task? This question will be discussed in the next section. We favor deconvolution based on the maximum entropy principle. It has been shown that this principle provides us with a method of inference that is both consistent and unbiased (in a mathematically well defined way). For a justification of this claim, we refer the reader to the material in Appendix A, and the papers referenced therein.

DECONVOLUTION AS A PROBLEM OF INFERENCE

In most cases, when we make a neutron spectrometry measurement, we have some information about both the neutron source and about the physics that governs the interactions of the neutrons as they travel from the source to the spectrometer. This information is usually derived from calculations or independent measurements. As we stressed in the introduction, we need to incorporate all this *a priori* information into the deconvolution process. We refer to this initial estimate of the spectrum that incorporates all the *a priori* information as the default spectrum.

Suppose that we have chosen a default spectrum that incorporates the *a priori* information that is available before the measurements. How do we modify that default spectrum to take into

account the new information contained in our measurements? To answer this question, note that a neutron spectrum is a positive distribution that can be interpreted as a probability distribution (in the following sense: asked for the probability that the next incoming neutron is in a given energy range, all you need to do is look up the neutron energy spectrum, and the fraction of neutrons in that energy range is the answer). This permits us to reformulate the question in the following way: given a prior probability distribution (the default spectrum), which method should we use to derive a new probability distribution (the spectrum which results from the deconvolution) that takes into account the new information provided by our measurements? A formal argument [due to Shore and Johnson (1980); see Appendix A for a brief discussion] shows that the maximum entropy method is the only general method of choosing the new distribution that does not lead to inconsistencies. A derivation of this important result is beyond the scope of this paper, and we refer the reader to the original article for a proof. Instead, we will try to justify the use of the maximum entropy principle using more intuitive arguments.

The default spectrum contains all the *a priori* information available. Therefore, any deviations from the default spectrum that results from the deconvolution should be driven by the new information provided by the measurements, otherwise we would be introducing structure that neither agrees with our *a priori* information nor is justified by the measurements. In other words, we want to take our default spectrum, and change it into a spectrum that fits the data but remains "as close as possible" to the default spectrum. But what does "as close as possible" mean when comparing two spectra? Once more, we interpret spectra in terms of probability distributions. Given a prior distribution, p(x), and a distribution, q(x), we define the cross-entropy, S_{CE}, by

$$S_{CE} = - \int \{q(x) \ln (q(x)/p(x))\} \, dx. \qquad (2.1)$$

The cross-entropy satisfies $S_{CE} \leq 0$, and it equals zero only if q(x) = p(x). It can be argued (Hobson 1969) that the magnitude $|S_{CE}|$ of the cross-entropy provides a measure of the amount of information necessary to change p(x), which by assumption contains the *a priori* information, into q(x). It seems reasonable, therefore, to use $|S_{CE}|$ as a measure of how much q(x) differs from p(x). From among all the spectra that fit the data, the maximum entropy method chooses the one for which $|S_{CE}|$ is a minimum, and, therefore, the one that is "closest" to the default spectrum.

MAXIMUM ENTROPY DECONVOLUTION OF MULTISPHERE NEUTRON SPECTROMETER DATA

We now describe the main features of the computer program MAXED (maximum entropy deconvolution) that we developed to apply the maximum entropy method to the deconvolution of multisphere neutron spectrometer data. Note that the experimental errors for each individual detector are taken into account during deconvolution, which is essential for the proper analysis of the data. A user's guide with practical recommendations for running the code can be found in Appendix B.

In practice, the deconvolution problem will usually be formulated in discrete terms. To do this for a multisphere neutron spectrometer with m detectors, we introduce an energy bin structure with energy bins ΔE_i (i = 1, ..., n and n > m), and replace the spectrum f(E) and the response functions $R_k(E)$ that appear in Equation (1.1) with a (discretized) spectrum f_i and (discretized) response functions R_{ki}. We define the set of admissible spectra using two restrictions:

$$N_k + \epsilon_k = \sum R_{ki} f_i, \qquad k = 1, ..., m \tag{3.1}$$

$$\sum \epsilon_k^2/\sigma_k^2 = \Omega \tag{3.2}$$

where the N_k are the measurements, σ_k the standard deviations, and the left hand of equation (3.2) is the familiar chi-square statistic. Typically, Ω is set equal to the number of detectors. The set of Equations (3.1) is the discretized version of (1.1). Equation (3.2) is a constraint for handling the (unknown) errors ϵ_k, and assumes that the errors are normally distributed with zero mean and variances σ_k^2. From this set of admissible spectra, we want to select the one that maximizes the entropy S of the distribution,

$$S = - \sum \{f_i \ln (f_i/f_i^{DEF}) + f_i^{DEF} - f_i \} \tag{3.3}$$

where the f_i^{DEF} is the (discretized) default spectra. Equation (3.3) is in the form given in Skilling (1989). For the relationship of S to S_{CE}, see Appendix A and the references cited.

We now present the maximum entropy deconvolution algorithm, which is a modification of the one in Wilczek and Drapatz (1985).

The Lagrangian associated with the maximization of (3.3) with constraints (3.1) and (3.2) is of the form

$$L(f_i, \epsilon_k, \lambda_k, \mu) = -\sum \{f_i \ln (f_i/f_i^{DEF}) + f_i^{DEF} - f_i\} - \sum \lambda_k \{\sum R_{ki} f_i - N_k - \epsilon_k\} - \mu \{\sum(\epsilon_k/\sigma_k)^2 - \Omega\}. \quad (3.4)$$

where λ_k, μ are (m + 1) Lagrange multipliers. Variation with respect to f_i, e_k and μ lead to the set of (n + m + 1) equations

$$-\ln (f_i/f_i^{DEF}) - \sum \lambda_k R_{ki} = 0, \qquad i = 1, ..., n \quad (3.5)$$

$$\lambda_k - 2\mu\epsilon_k/\sigma_k^2 = 0, \qquad k = 1, ..., m \quad (3.6)$$

$$\sum \epsilon_k^2/\sigma_k^2 = \Omega. \quad (3.7)$$

We use (3.5), (3.6) and (3.7) to solve for the f_i, the ϵ_k and μ in terms of the λ_k,

$$f_i = f_i^{DEF} \exp\{-\sum\lambda_k R_{ki}\} \qquad i = 1, ..., n \quad (3.8)$$

$$\epsilon_k = \lambda_k\sigma_k^2/2\mu, \qquad k = 1, ..., m \quad (3.9)$$

$$\mu = (\sum (\lambda_k\sigma_k)^2/4\Omega)^{1/2}. \quad (3.10)$$

Variation with respect to the λ_k leads to (3.1), and using (3.8), (3.9), and (3.10) we can write these m equations as

$$N_k + \lambda_k \sigma_k^2 \left(\Omega / \sum (\lambda_j \sigma_j)^2 \right)^{1/2} - \sum R_{kj} f_j^{DEF} \exp\{-\sum \lambda_l R_{lj}\} = 0, \; k = 1, ..., m. \quad (3.11)$$

Thus, the initial optimization problem stated has been reduced to a system of m equations with m unknowns $\lambda_1, ..., \lambda_m$. The set of equations (3.11) can also be derived by maximizing the potential function, Z,

$$Z = - \sum f_i^{DEF} \exp\{-\sum \lambda_k R_{ki}\} - (\Omega \sum (\lambda_k \sigma_k)^2)^{1/2} - \sum N_k \lambda_k \quad (3.12)$$

with respect to the λ_k. Therefore, we can reformulate the problem in terms of the maximization of Z.

We use a simulated annealing algorithm to maximize the potential function (3.12). Simulated annealing is a global optimization method that distinguishes between different local optima [see Press et al. (1992) for a brief discussion]. The algorithm makes very few assumptions regarding the function to be optimized, and it is quite robust with respect to non-quadratic surfaces. It moves both up and downhill (and can, therefore, escape from local optimas), and as the optimization process proceeds, it focuses on the most promising area. The method is, therefore, able to discriminate between "gross behavior" and finer "wrinkles." Our simulated annealing subroutine is a modification of the program SIMANN.F. This program is an example of the Corana et al. (1987) simulated annealing algorithm for multimodal and robust optimization, as implemented and modified by Goffe et al. (1994).

To start the optimization, the algorithm randomly chooses a trial point within a given step length of the user selected starting point. The function is evaluated at this trial point and its value is compared to its value at the initial point. In a maximization problem, all uphill moves are accepted and the algorithm continues from that trial point. Downhill moves may be accepted, and the decision is made in a probabilistic manner by the Metropolis criteria, which is based on an analogy with thermodynamics -- specifically, the way metals cool and anneal [for a discussion of the Metropolis criteria see, for example, Press et al. (1992)]. If the new point is rejected, another point is chosen instead for a trial evaluation. As the optimization process proceeds, the length of the steps is reduced and the algorithm closes in on the global optimum.

To initialize the parameters of the simulated annealing algorithm we took into consideration the values suggested in Corana et al. (1987) and Goffe et al. (1994), and used the results of test runs to find a set of optimal initial values. The only parameter that we did not initialize in this way was the temperature (a parameter which enters into the Metropolis criteria), because the optimal initial value of the temperature depends, in general, on the particular optimization problem that is being solved. In most cases, an initial value for the temperature can be chosen as follows. Note that simulated annealing first builds up a rough view of the surface by moving with large step lengths at a high temperature. In our case, the function is convex in the large, and has only one maximum at a large enough scale length. Therefore, choose an initial temperature that is high enough so that no new maxima are found in the first set of iterations at constant temperature. This corresponds to probing the surface to within a scale length that is so large that we already are at the global maximum for the given scale length. Then, as the temperature is decreased, the algorithm is given the chance to inspect the "fine wrinkles" of the function at temperatures that are still high enough to escape from any of the local maxima. After searching through the local maxima, the algorithm should arrive at the global maximum.

AN APPLICATION OF THE DECONVOLUTION METHOD

We now describe an application of the code MAXED in which we analyzed measurements taken 120 m from the center of the Tokamak Fusion Test Reactor (TFTR) of the Princeton Plasma Physics Laboratory. In this example, we used the multisphere neutron spectrometer described in the Introduction to measure neutrons generated during deuterium-tritium (D-T) fusion reactions, and, from these data, we estimated the spectrum for neutrons with energies ranging from epithermal to 14 MeV. The data analyzed here were collected in 1996 using a smaller set of eight detectors, corresponding to response functions 3, 5, and 7 through 12 of Figure 1. We did not use detectors 1 and 2, which are normally used to infer the thermal component of the spectrum. For the deconvolution, the σ_k were set equal to the square roots of the measured number of counts N_k, and ranged from 2.5% to 5% of the N_k. We estimate that these relatively large statistical errors dominated other sources of error.

The 14 MeV neutrons generated during fusion reactions travel through a substantial amount of shielding before reaching the spectrometer. It is exceedingly difficult to create a detailed model of the shielding at TFTR. Fusion reactor shielding simulations, especially those for neutrons, are subject to uncertainties due to homogenized approximations of complex

geometries, uncertainties in materials properties and distributions, and changes in configuration. Most of the shielding is due to approximately one and a half meters of concrete in the walls and the ceiling. We have used, therefore, a default spectrum derived from a calculation of 14 MeV neutrons incident on a 1.54 m thick slab of type 4 concrete (Ku and Liew 1992). For the deconvolution, we re-binned the default spectrum to fit the bin structure of the response functions (20 bins per decade, of equal width in the logarithm of the energy) using linear interpolation in the logarithm of the energy.

Figure 2 shows both the default spectrum and the maximum entropy spectrum that results from the deconvolution. We can use the chi-square statistic as an indicator of how well the spectra fit the data. The chi-square of the maximum entropy spectrum is 8, which equals the number of detectors (this was one of the two restrictions used to define the set of admissible spectra, see Equation 3.2). The maximum entropy spectrum, therefore, fits the data within experimental error. The default spectrum has a higher chi-square of 262.

A priori structure that is consistent with the data is preserved by the deconvolution method. The fine structure in the default spectrum (due to the physics of the transport of neutrons through the shielding) appears in the maximum entropy spectrum, even though the peaks are sharper than the resolution of the spectrometer. The overall spectral shape has changed, however, to account for fewer higher energy neutrons: below approximately 0.4 MeV the maximum entropy spectrum is above the default spectrum, otherwise the maximum entropy spectrum is lower than the default spectrum. Features of the default spectrum that are not consistent with the data are, therefore, corrected. Some small features in the maximum entropy spectrum seem to be due to structure in the response functions. If we re-bin the maximum entropy spectrum so that its bin structure matches the bin structure of default spectrum, the chi-square increases only slightly (from 8.0 to 8.1), indicating that the added structure is very minor.

DISCUSSION

The problem of deriving the neutron spectrum from a set of multisphere neutron spectrometer measurements has no unique solution, and many different spectra are consistent with the data. It follows that the neutron spectrum cannot be derived from the measurements alone, and additional conditions need to be introduced to choose one spectrum from all the spectra that fit the data. We are faced then with a problem of *inference*, in which we want to

provide our best estimate of the spectrum given all the information that is available, and not only the measurements. This is an important point. In general, it has not been recognized that the deconvolution of multisphere neutron spectrometer measurements should be treated as a problem of inference, but without taking this into consideration it is difficult to choose a deconvolution method among the many that are available. As a result, in many cases data are analyzed using deconvolution methods that are not correct, either because they do not take into account *a priori* information in a systematic way (some even claim not to use *a priori* information at all!), or because they do not incorporate the new information derived from the measurements in a consistent way. The most popular deconvolution methods are iterative algorithms which require a starting spectrum. While one can argue that the starting spectrum can be used to encode *a priori* information, in general, the algorithms used to derive the final spectrum cannot be justified when examined as methods of inference.

We have argued in this paper that the maximum entropy method is theoretically sound, and that it should be used for the deconvolution of multisphere neutron spectrometer measurements. The code MAXED described here provides a numerical realization of this method. Although the conceptual difficulties associated with deconvolution are solved when the problem is approached from this point of view, we want to point out that one difficult problem remains -- that of the default spectrum. More attention needs to be devoted to constructing good default spectra from the *a priori* information available.

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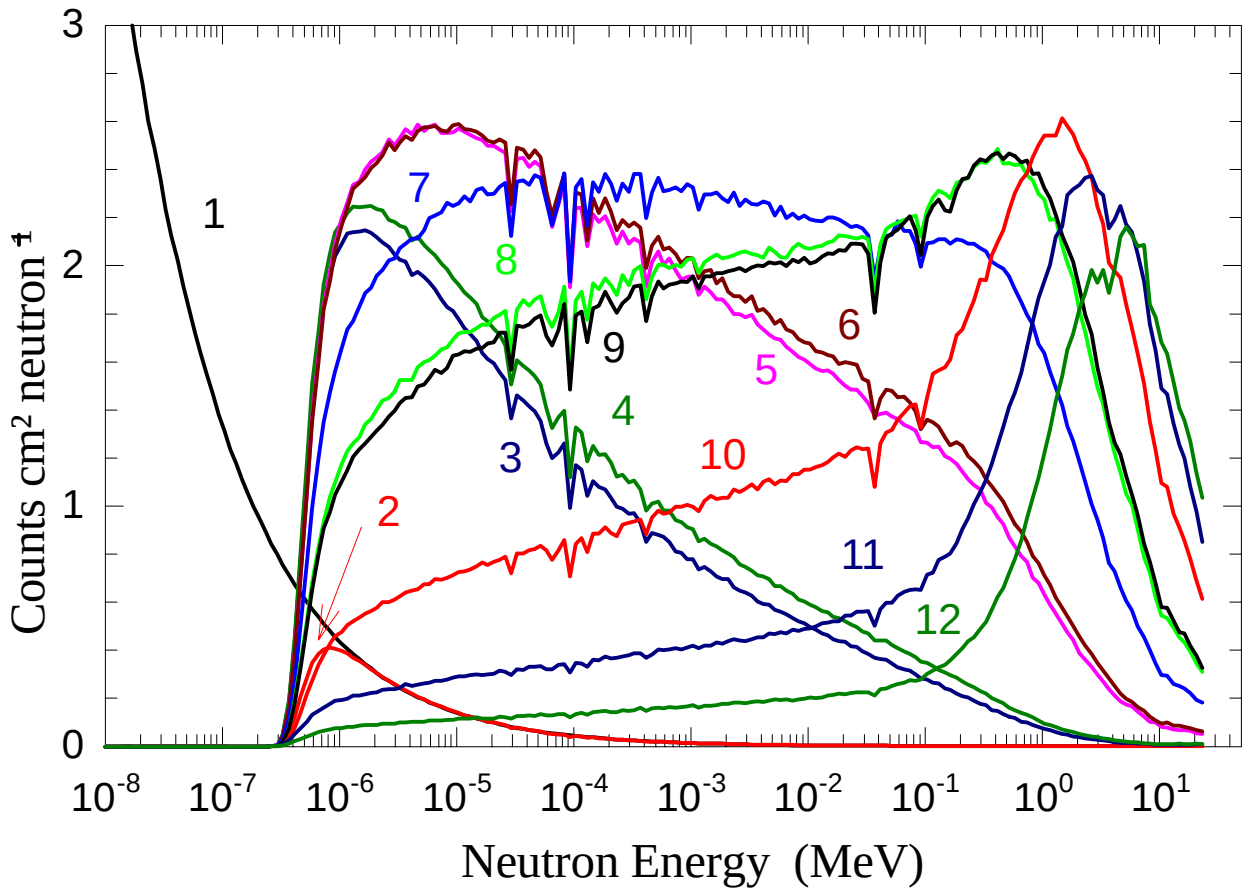

Figure 1. Calculated response functions of EML multisphere neutron spectrometer detectors with BF_3 counters.

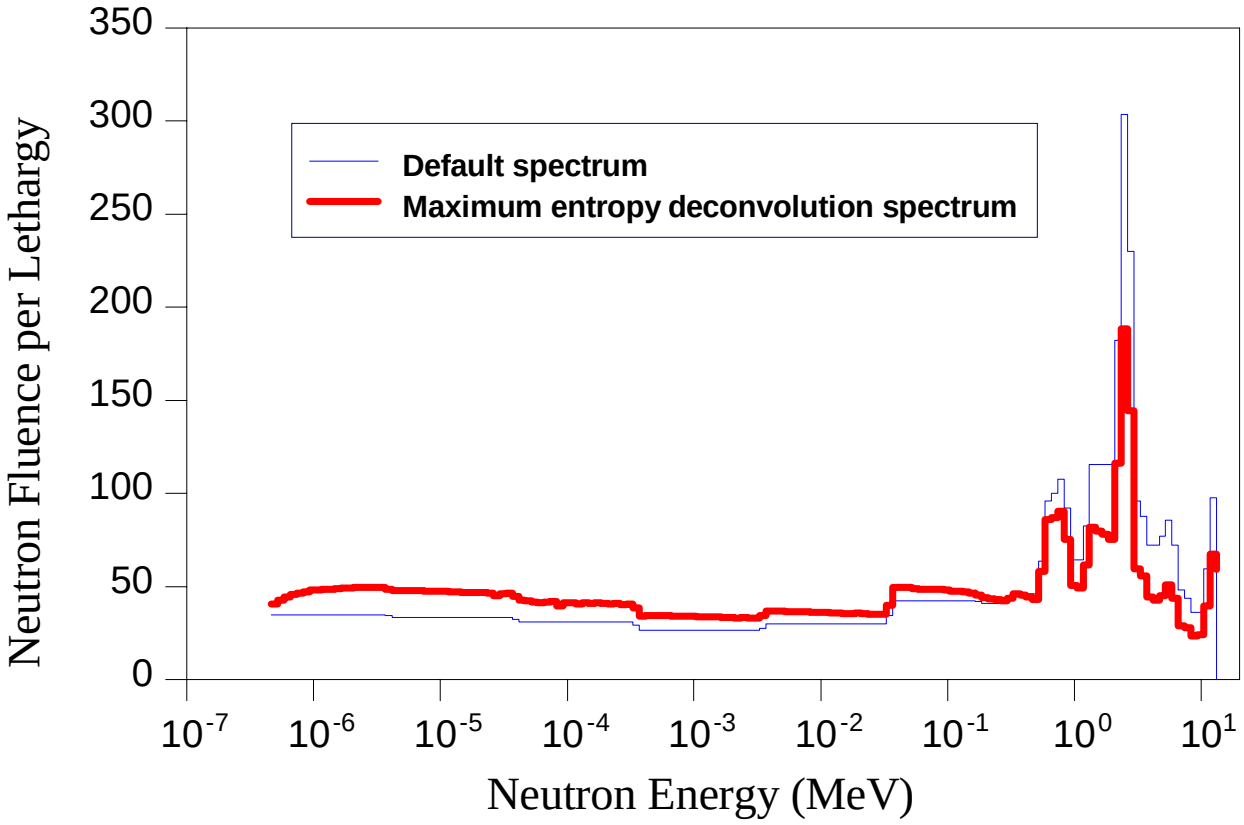

Figure 2. Default spectrum and maximum entropy deconvolution spectrum for the example discussed in **An Application of the Deconvolution Method**.

Figure 3. Response function file of the example discussed in **An Application of the Deconvolution Method**.

```
12
228
cm**2
1.0416400e-10
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9.2836659e-10,8.58685E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.0416445e-09,8.31688E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.1687443e-09,8.03437E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.3113526e-09,7.79806E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.4713619e-09,7.57419E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.6458952e-09,7.29318E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.8473349e-09,7.07512E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.0783539e-09,6.76614E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.3319515e-09,6.52769E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.6164925e-09,6.30414E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.9357529e-09,6.06056E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
3.2939689e-09,5.80649E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
3.6958940e-09,5.55589E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
4.1468612e-09,5.32749E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
4.6528548e-09,5.10289E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
5.2205890e-09,4.90448E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
5.8575972e-09,4.67246E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
6.5723321e-09,4.47370E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
7.3742779e-09,4.26611E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
8.2740759e-09,4.07584E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
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Figure 3 (Cont'd)

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9.2836659e-09,3.88518E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.0416445e-08,3.70402E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.1687443e-08,3.53473E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.3113526e-08,3.36666E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.4713619e-08,3.20097E+00,1.84224E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.6458952e-08,3.04656E+00,2.38625E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.8473349e-08,2.89673E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.0783539e-08,2.75642E+00,8.48560E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.3319515e-08,2.59873E+00,1.09095E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.6164925e-08,2.48434E+00,6.40621E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.9357529e-08,2.36360E+00,1.30726E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
3.2939689e-08,2.23466E+00,1.62426E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
3.6958940e-08,2.12197E+00,1.03654E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
4.1468612e-08,2.02174E+00,1.49856E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
4.6528548e-08,1.91278E+00,7.65304E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
5.2205890e-08,1.81736E+00,1.14813E-04,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
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6.5723321e-08,1.63271E+00,5.76141E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
7.3742779e-08,1.54266E+00,4.90013E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
8.2740759e-08,1.46409E+00,1.88486E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
9.2836659e-08,1.38191E+00,1.44918E-05,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.0416445e-07,1.31392E+00,3.50964E-06,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.1687443e-07,1.23736E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.3113526e-07,1.17298E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.4713619e-07,1.11454E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.6458952e-07,1.05455E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
1.8473349e-07,1.00014E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.0783539e-07,9.45492E-01,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.3319515e-07,8.91031E-01,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00,0.00000E+00
2.6164926e-07,8.42393E-01,4.55849E-05,1.14704E-04,1.12810E-04,1.15428E-04,1.93666E-04,5.02245E-05,1.48150E-04,2.60294E-05,1.11138E-05,2.62826E-04,7.30684E-05
2.9357529e-07,7.98321E-01,1.54470E-03,4.58638E-03,3.76505E-03,4.08332E-03,3.58394E-03,2.57076E-03,1.59944E-03,1.68806E-03,5.27996E-04,1.97913E-03,7.53260E-04
3.2939689e-07,7.55176E-01,1.36806E-02,4.54893E-02,4.72781E-02,4.06326E-02,4.16951E-02,2.95916E-02,1.79858E-02,1.69583E-02,7.91942E-03,9.36362E-03,3.84506E-03
3.6958940e-07,7.13331E-01,5.34337E-02,1.84366E-01,1.92096E-01,1.58544E-01,1.57910E-01,1.15418E-01,7.52353E-02,7.41172E-02,3.11615E-02,2.59899E-02,1.06154E-02
4.1468612e-07,6.72762E-01,1.22997E-01,4.39237E-01,4.58320E-01,3.91461E-01,3.90424E-01,2.74447E-01,1.93794E-01,1.80040E-01,7.87552E-02,5.11939E-02,2.12553E-02
4.6528548e-07,6.37347E-01,2.07699E-01,7.73124E-01,8.10226E-01,7.07727E-01,7.08572E-01,5.03970E-01,3.47814E-01,3.26322E-01,1.40945E-01,7.63909E-02,3.07345E-02
5.2205890e-07,6.00976E-01,2.85945E-01,1.13200E+00,1.18169E+00,1.05639E+00,1.05901E+00,7.66373E-01,5.24945E-01,5.04022E-01,2.18213E-01,1.02486E-01,4.14578E-02
5.8575972e-07,5.70219E-01,3.48711E-01,1.43953E+00,1.50624E+00,1.37010E+00,1.34923E+00,9.91916E-01,6.80640E-01,6.50625E-01,2.72897E-01,1.31096E-01,4.99081E-02
6.5723321e-07,5.38696E-01,3.83159E-01,1.67132E+00,1.72334E+00,1.62398E+00,1.59891E+00,1.20013E+00,8.29752E-01,7.99552E-01,3.37604E-01,1.48224E-01,5.97684E-02
7.3742779e-07,5.08822E-01,4.02532E-01,1.85939E+00,1.91964E+00,1.83064E+00,1.81774E+00,1.34656E+00,9.33003E-01,9.04961E-01,3.79836E-01,1.61759E-01,6.40300E-02
8.2740759e-07,4.80718E-01,4.09134E-01,1.95884E+00,2.05068E+00,1.97655E+00,1.93618E+00,1.47285E+00,1.03749E+00,9.75863E-01,4.26200E-01,1.75272E-01,7.04039E-02
9.2836658e-07,4.54450E-01,4.05593E-01,2.03968E+00,2.13926E+00,2.09093E+00,2.06739E+00,1.56051E+00,1.09706E+00,1.04544E+00,4.58276E-01,1.90515E-01,7.50186E-02
1.0416445e-06,4.28680E-01,3.97009E-01,2.08817E+00,2.17879E+00,2.19543E+00,2.16545E+00,1.65401E+00,1.17238E+00,1.09934E+00,4.73899E-01,1.91473E-01,7.46739E-02
```

Figure 3 (Cont'd)

1.1687443e-06,4.05470E-01,3.85441E-01,2.13386E+00,2.21798E+00,2.26056E+00,2.24065E+00,1.73375E+00,1.22872E+00,1.14898E+00,5.05100E-01,1.96683E-01,8.16591E-02
1.3113526e-06,3.83234E-01,3.70032E-01,2.13796E+00,2.24646E+00,2.33705E+00,2.32063E+00,1.76916E+00,1.25967E+00,1.20419E+00,5.19862E-01,2.10752E-01,8.23749E-02
1.4713619e-06,3.62856E-01,3.55282E-01,2.14086E+00,2.24368E+00,2.35024E+00,2.34395E+00,1.82989E+00,1.28592E+00,1.23787E+00,5.35935E-01,2.13462E-01,8.40999E-02
1.6458952e-06,3.42145E-01,3.38084E-01,2.14629E+00,2.24458E+00,2.40024E+00,2.36524E+00,1.88585E+00,1.33848E+00,1.26001E+00,5.50973E-01,2.16626E-01,8.92774E-02
1.8473349e-06,3.24019E-01,3.21855E-01,2.13500E+00,2.24794E+00,2.43261E+00,2.40640E+00,1.90820E+00,1.36364E+00,1.28806E+00,5.62490E-01,2.21237E-01,8.85770E-02
2.0783539e-06,3.05630E-01,3.05370E-01,2.11645E+00,2.23424E+00,2.45071E+00,2.42683E+00,1.95805E+00,1.40075E+00,1.31313E+00,5.75589E-01,2.26014E-01,8.99447E-02
2.3319514e-06,2.87974E-01,2.88333E-01,2.09682E+00,2.22921E+00,2.47292E+00,2.45922E+00,1.98360E+00,1.43934E+00,1.33795E+00,6.00501E-01,2.35447E-01,9.32261E-02
2.6164925e-06,2.72269E-01,2.72831E-01,2.07605E+00,2.19016E+00,2.52667E+00,2.50915E+00,2.02139E+00,1.44830E+00,1.37325E+00,6.00667E-01,2.38578E-01,9.31824E-02
2.9357529e-06,2.56563E-01,2.58068E-01,2.04612E+00,2.18105E+00,2.50226E+00,2.48078E+00,2.03176E+00,1.47452E+00,1.39614E+00,6.09968E-01,2.39056E-01,9.31368E-02
3.2939689e-06,2.43170E-01,2.45100E-01,2.02476E+00,2.16576E+00,2.53162E+00,2.51215E+00,2.07271E+00,1.52436E+00,1.42733E+00,6.24903E-01,2.41524E-01,9.87025E-02
3.6958940e-06,2.29363E-01,2.31327E-01,2.01637E+00,2.15667E+00,2.56644E+00,2.53994E+00,2.09164E+00,1.52550E+00,1.44949E+00,6.41687E-01,2.57096E-01,9.81618E-02
4.1468612e-06,2.17363E-01,2.19391E-01,1.95935E+00,2.10669E+00,2.54550E+00,2.52144E+00,2.11147E+00,1.52364E+00,1.45211E+00,6.48883E-01,2.51511E-01,1.03573E-01
4.6528548e-06,2.04517E-01,2.06449E-01,1.97414E+00,2.11984E+00,2.58645E+00,2.56074E+00,2.11457E+00,1.57813E+00,1.49550E+00,6.53732E-01,2.55234E-01,1.01060E-01
5.2205890e-06,1.93087E-01,1.94970E-01,1.95320E+00,2.08603E+00,2.55854E+00,2.57436E+00,2.14713E+00,1.59996E+00,1.50464E+00,6.73943E-01,2.61052E-01,1.00477E-01
5.8575971e-06,1.82557E-01,1.84326E-01,1.93020E+00,2.06454E+00,2.57889E+00,2.57532E+00,2.19696E+00,1.60748E+00,1.53258E+00,6.75413E-01,2.63242E-01,1.07449E-01
6.5723321e-06,1.72607E-01,1.74615E-01,1.90233E+00,2.04277E+00,2.58510E+00,2.58139E+00,2.20954E+00,1.62387E+00,1.56566E+00,6.95833E-01,2.73026E-01,1.07968E-01
7.3742779e-06,1.63423E-01,1.65007E-01,1.85838E+00,2.00648E+00,2.55177E+00,2.56331E+00,2.22091E+00,1.65318E+00,1.55359E+00,6.93588E-01,2.74169E-01,1.07876E-01
8.2740759e-06,1.53891E-01,1.55700E-01,1.82762E+00,1.98417E+00,2.55067E+00,2.55758E+00,2.26156E+00,1.66179E+00,1.57318E+00,7.00243E-01,2.78687E-01,1.11537E-01
9.2836659e-06,1.45076E-01,1.46820E-01,1.81385E+00,1.94687E+00,2.55597E+00,2.58277E+00,2.24962E+00,1.69172E+00,1.62482E+00,7.13989E-01,2.85068E-01,1.10682E-01
1.0416445e-05,1.37867E-01,1.39364E-01,1.77557E+00,1.92137E+00,2.56935E+00,2.58842E+00,2.25891E+00,1.71745E+00,1.63029E+00,7.25802E-01,2.91923E-01,1.14904E-01
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2.0783539e-05,9.73859E-02,9.81222E-02,1.59324E+00,1.73740E+00,2.49624E+00,2.50962E+00,2.29758E+00,1.78238E+00,1.67906E+00,7.68857E-01,3.02760E-01,1.16932E-01
2.3319515e-05,9.18517E-02,9.28319E-02,1.55533E+00,1.71257E+00,2.48231E+00,2.52606E+00,2.32767E+00,1.79465E+00,1.71860E+00,7.74006E-01,3.14256E-01,1.21655E-01
2.6164925e-05,8.69970E-02,8.78201E-02,1.52419E+00,1.66982E+00,2.46925E+00,2.51261E+00,2.34580E+00,1.81278E+00,1.71872E+00,7.89495E-01,3.09690E-01,1.19798E-01
2.9357529e-05,8.19604E-02,7.49621E-02,1.36553E+00,1.50433E+00,2.22571E+00,2.25475E+00,2.12177E+00,1.63631E+00,1.56555E+00,7.19253E-01,2.97299E-01,1.14019E-01
3.2939689e-05,7.74218E-02,7.83241E-02,1.46011E+00,1.60704E+00,2.44375E+00,2.48913E+00,2.35039E+00,1.81701E+00,1.75095E+00,8.02903E-01,3.21522E-01,1.27975E-01
3.6958940e-05,7.28283E-02,7.36206E-02,1.43933E+00,1.58372E+00,2.44752E+00,2.48258E+00,2.35462E+00,1.85697E+00,1.74557E+00,8.02532E-01,3.18929E-01,1.26940E-01
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5.8575971e-05,5.78326E-02,5.59351E-02,1.26187E+00,1.40724E+00,2.27858E+00,2.30906E+00,2.26778E+00,1.78942E+00,1.70635E+00,8.15346E-01,3.19440E-01,1.30056E-01
6.5723321e-05,5.45479E-02,5.02766E-02,1.19882E+00,1.32447E+00,2.16109E+00,2.20823E+00,2.16927E+00,1.74438E+00,1.66766E+00,7.76783E-01,3.19687E-01,1.30389E-01
7.3742779e-05,5.15817E-02,4.95642E-02,1.22327E+00,1.36994E+00,2.25539E+00,2.30040E+00,2.26107E+00,1.80614E+00,1.73132E+00,8.14577E-01,3.37162E-01,1.35530E-01
8.2740759e-05,4.85459E-02,4.90289E-02,1.26010E+00,1.39681E+00,2.33683E+00,2.38153E+00,2.38407E+00,1.91106E+00,1.84060E+00,8.59717E-01,3.42178E-01,1.35392E-01
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1.0416445e-04,4.33281E-02,4.27849E-02,1.16862E+00,1.32703E+00,2.24121E+00,2.30318E+00,2.32595E+00,1.85467E+00,1.79372E+00,8.42284E-01,3.46949E-01,1.35409E-01
1.1687443e-04,4.10016E-02,4.06293E-02,1.15249E+00,1.30599E+00,2.23936E+00,2.29575E+00,2.35980E+00,1.89234E+00,1.80761E+00,8.66193E-01,3.44811E-01,1.39928E-01
1.3113526e-04,3.87366E-02,3.57367E-02,1.04140E+00,1.18409E+00,2.08026E+00,2.10554E+00,2.16564E+00,1.77304E+00,1.68075E+00,8.07773E-01,3.28410E-01,1.31190E-01

Figure 3 (Cont'd)

1.4713619e-04,3.66072E-02,3.65196E-02,1.10312E+00,1.25049E+00,2.21246E+00,2.27928E+00,2.34709E+00,1.92639E+00,1.82246E+00,8.83808E-01,3.59007E-01,1.42821E-01
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2.0783539e-04,3.06379E-02,3.05675E-02,1.03393E+00,1.17993E+00,2.15388E+00,2.22467E+00,2.34055E+00,1.91650E+00,1.84414E+00,8.87465E-01,3.65397E-01,1.41823E-01
2.3319515e-04,2.88174E-02,2.79040E-02,9.97679E-01,1.12032E+00,2.09163E+00,2.14583E+00,2.26953E+00,1.87810E+00,1.80500E+00,8.74530E-01,3.70031E-01,1.41597E-01
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4.1468612e-03,6.70366E-03,6.73333E-03,5.92975E-01,7.04554E-01,1.73501E+00,1.82146E+00,2.26693E+00,2.03310E+00,1.99778E+00,1.09993E+00,4.54764E-01,1.83442E-01
4.6528548e-03,6.36097E-03,6.38885E-03,5.87984E-01,6.95722E-01,1.71243E+00,1.79562E+00,2.25792E+00,2.07406E+00,2.01260E+00,1.11211E+00,4.70124E-01,1.88697E-01
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7.3742779e-03,4.99826E-03,5.01961E-03,5.32943E-01,6.34918E-01,1.63911E+00,1.72535E+00,2.21546E+00,2.07115E+00,2.02551E+00,1.13184E+00,4.84089E-01,1.90911E-01
8.2740759e-03,4.71103E-03,4.74500E-03,5.20391E-01,6.11466E-01,1.62027E+00,1.69502E+00,2.19600E+00,2.09390E+00,2.01273E+00,1.13293E+00,4.80519E-01,1.94527E-01
9.2836658e-03,4.48389E-03,4.50999E-03,5.12045E-01,6.03903E-01,1.60852E+00,1.68093E+00,2.20800E+00,2.07390E+00,2.03232E+00,1.15355E+00,4.88600E-01,1.96426E-01
1.0416445e-02,4.18655E-03,4.21850E-03,4.95934E-01,5.89491E-01,1.58937E+00,1.67672E+00,2.19223E+00,2.07006E+00,2.00810E+00,1.15152E+00,4.90468E-01,2.02076E-01
1.1687443e-02,3.96537E-03,3.99088E-03,4.84589E-01,5.80109E-01,1.57044E+00,1.64356E+00,2.18807E+00,2.07663E+00,2.01874E+00,1.16125E+00,4.98810E-01,1.99403E-01
1.3113526e-02,3.74006E-03,3.76630E-03,4.70147E-01,5.63786E-01,1.56161E+00,1.63785E+00,2.18481E+00,2.08218E+00,2.03821E+00,1.16937E+00,5.12596E-01,2.03882E-01
1.4713619e-02,3.53616E-03,3.56296E-03,4.62631E-01,5.56457E-01,1.56363E+00,1.62866E+00,2.20162E+00,2.09519E+00,2.05140E+00,1.18369E+00,5.13572E-01,2.12005E-01
1.6458952e-02,3.33767E-03,3.36365E-03,4.46527E-01,5.38783E-01,1.54845E+00,1.62278E+00,2.20061E+00,2.10057E+00,2.04804E+00,1.20167E+00,5.20399E-01,2.14903E-01

Figure 3 (Cont'd)

1.8473349e-02,3.14810E-03,3.17553E-03,4.38036E-01,5.33992E-01,1.52206E+00,1.59134E+00,2.19371E+00,2.12580E+00,2.06569E+00,1.21762E+00,5.20088E-01,2.19228E-01
2.0783539e-02,2.97884E-03,3.00668E-03,4.34298E-01,5.19207E-01,1.49802E+00,1.59936E+00,2.17710E+00,2.12168E+00,2.06005E+00,1.20284E+00,5.43401E-01,2.17401E-01
2.3319514e-02,2.79861E-03,2.82504E-03,4.16886E-01,5.07076E-01,1.48402E+00,1.59284E+00,2.16124E+00,2.11664E+00,2.09118E+00,1.23147E+00,5.42713E-01,2.20467E-01
2.6164926e-02,2.64113E-03,2.66686E-03,4.11972E-01,4.95762E-01,1.48453E+00,1.58739E+00,2.16624E+00,2.11326E+00,2.09072E+00,1.24438E+00,5.58412E-01,2.26506E-01
2.9357529e-02,2.46258E-03,2.48353E-03,3.94936E-01,4.77314E-01,1.45899E+00,1.52932E+00,2.15360E+00,2.11935E+00,2.08827E+00,1.23686E+00,5.59878E-01,2.24491E-01
3.2939689e-02,2.38073E-03,2.40302E-03,3.84264E-01,4.69846E-01,1.42528E+00,1.51910E+00,2.12706E+00,2.11089E+00,2.04838E+00,1.24091E+00,5.64204E-01,2.26278E-01
3.6958940e-02,2.24646E-03,2.26613E-03,3.68766E-01,4.39939E-01,1.37419E+00,1.36465E+00,1.94434E+00,1.88747E+00,1.80578E+00,1.08092E+00,5.01890E-01,2.12416E-01
4.1468612e-02,2.12541E-03,2.14655E-03,3.63038E-01,4.39891E-01,1.38365E+00,1.44756E+00,2.07861E+00,2.08783E+00,2.01413E+00,1.26007E+00,5.71917E-01,2.37269E-01
4.6528548e-02,2.00213E-03,2.02268E-03,3.60673E-01,4.41556E-01,1.38807E+00,1.48271E+00,2.12162E+00,2.15296E+00,2.11548E+00,1.30698E+00,5.95683E-01,2.40570E-01
5.2205890e-02,1.89642E-03,1.91815E-03,3.44461E-01,4.23383E-01,1.37212E+00,1.47826E+00,2.12827E+00,2.16808E+00,2.13211E+00,1.31661E+00,5.95109E-01,2.49430E-01
5.8575971e-02,1.79418E-03,1.81520E-03,3.27277E-01,4.07165E-01,1.36295E+00,1.45153E+00,2.18310E+00,2.17987E+00,2.14902E+00,1.36024E+00,6.25017E-01,2.59702E-01
6.5723321e-02,1.69541E-03,1.71367E-03,3.21155E-01,3.94496E-01,1.33780E+00,1.45396E+00,2.14202E+00,2.18956E+00,2.17944E+00,1.38834E+00,6.42626E-01,2.62423E-01
7.3742779e-02,1.61361E-03,1.63248E-03,3.14903E-01,3.83939E-01,1.32684E+00,1.43192E+00,2.14358E+00,2.19943E+00,2.20624E+00,1.41585E+00,6.49986E-01,2.76011E-01
8.2740759e-02,1.52685E-03,1.54360E-03,3.00627E-01,3.71024E-01,1.31348E+00,1.39305E+00,2.09170E+00,2.20967E+00,2.14582E+00,1.42160E+00,6.53057E-01,2.74163E-01
9.2836658e-02,1.45516E-03,1.47127E-03,2.83276E-01,3.51595E-01,1.26659E+00,1.31874E+00,1.99518E+00,2.12078E+00,2.04439E+00,1.32736E+00,6.50797E-01,2.78768E-01
1.0416445e-01,1.38245E-03,1.39877E-03,2.79402E-01,3.48710E-01,1.26281E+00,1.35818E+00,2.09482E+00,2.23236E+00,2.19905E+00,1.45174E+00,7.16728E-01,3.01754E-01
1.1687443e-01,1.31033E-03,1.32512E-03,2.62739E-01,3.32141E-01,1.24492E+00,1.34727E+00,2.09706E+00,2.28175E+00,2.24783E+00,1.55159E+00,7.23315E-01,3.14157E-01
1.3113526e-01,1.24095E-03,1.25557E-03,2.61812E-01,3.17588E-01,1.24789E+00,1.32835E+00,2.11068E+00,2.31352E+00,2.27551E+00,1.56443E+00,7.74433E-01,3.27922E-01
1.4713619e-01,1.17696E-03,1.18962E-03,2.40590E-01,3.06337E-01,1.20605E+00,1.28362E+00,2.10808E+00,2.29843E+00,2.22957E+00,1.57777E+00,7.90367E-01,3.38947E-01
1.6458952e-01,1.12254E-03,1.13553E-03,2.34340E-01,2.95319E-01,1.19474E+00,1.25199E+00,2.09004E+00,2.27957E+00,2.22489E+00,1.60827E+00,8.01140E-01,3.57864E-01
1.8473349e-01,1.05736E-03,1.06863E-03,2.19577E-01,2.85681E-01,1.17511E+00,1.26452E+00,2.10964E+00,2.34475E+00,2.31219E+00,1.71133E+00,8.44869E-01,3.84767E-01
2.0783539e-01,9.93837E-04,1.00421E-03,2.08566E-01,2.68047E-01,1.14438E+00,1.25033E+00,2.10306E+00,2.35228E+00,2.36092E+00,1.73479E+00,9.02222E-01,4.08231E-01
2.3319514e-01,9.25878E-04,9.34931E-04,2.03005E-01,2.56399E-01,1.11277E+00,1.20546E+00,2.09279E+00,2.36472E+00,2.36492E+00,1.79436E+00,9.49293E-01,4.37762E-01
2.6164926e-01,8.51528E-04,8.60945E-04,1.88626E-01,2.46065E-01,1.08810E+00,1.18402E+00,2.09286E+00,2.40975E+00,2.41551E+00,1.85578E+00,1.00835E+00,4.53036E-01
2.9357529e-01,7.73381E-04,7.82300E-04,1.76377E-01,2.29877E-01,1.06944E+00,1.15787E+00,2.07160E+00,2.43034E+00,2.44185E+00,1.94151E+00,1.05755E+00,4.87178E-01
3.2939689e-01,6.92471E-04,7.00199E-04,1.70455E-01,2.18258E-01,1.01264E+00,1.11274E+00,2.05984E+00,2.42632E+00,2.43407E+00,1.93715E+00,1.10809E+00,5.18853E-01
3.6958940e-01,6.24751E-04,6.31654E-04,1.50702E-01,2.01984E-01,9.93052E-01,1.08298E+00,2.01770E+00,2.45059E+00,2.45834E+00,2.04375E+00,1.17426E+00,5.68808E-01
4.1468612e-01,5.75630E-04,5.82345E-04,1.46916E-01,1.89897E-01,9.54262E-01,1.05002E+00,1.99929E+00,2.48314E+00,2.47088E+00,2.09686E+00,1.25190E+00,6.20509E-01
4.6528548e-01,5.54597E-04,5.61100E-04,1.35799E-01,1.78066E-01,9.23225E-01,1.01841E+00,1.98756E+00,2.42386E+00,2.44168E+00,2.14679E+00,1.32891E+00,6.59614E-01
5.2205890e-01,5.13731E-04,5.20062E-04,1.21770E-01,1.61879E-01,8.67252E-01,9.69088E-01,1.95574E+00,2.42669E+00,2.46898E+00,2.23046E+00,1.41179E+00,7.21423E-01
5.8575971e-01,4.28119E-04,4.33626E-04,1.17883E-01,1.51146E-01,8.30123E-01,9.23770E-01,1.90143E+00,2.41737E+00,2.45748E+00,2.30025E+00,1.47106E+00,7.78198E-01
6.5723321e-01,3.30518E-04,3.35111E-04,1.02966E-01,1.39866E-01,7.92090E-01,8.93650E-01,1.84869E+00,2.42026E+00,2.42738E+00,2.35325E+00,1.55432E+00,8.56686E-01
7.3742779e-01,2.52168E-04,2.57407E-04,9.86118E-02,1.32528E-01,7.66102E-01,8.48119E-01,1.79484E+00,2.37731E+00,2.43502E+00,2.38413E+00,1.65965E+00,9.49762E-01
8.2740759e-01,1.97199E-04,2.02432E-04,8.89287E-02,1.18077E-01,7.01292E-01,7.91904E-01,1.75657E+00,2.35542E+00,2.38021E+00,2.45227E+00,1.76660E+00,1.03414E+00
9.2836659e-01,1.60060E-04,1.64882E-04,8.07598E-02,1.07281E-01,6.79610E-01,7.60228E-01,1.67444E+00,2.29872E+00,2.38539E+00,2.50110E+00,1.86493E+00,1.11733E+00
1.0416445e+00,1.38734E-04,1.42531E-04,7.24933E-02,9.56116E-02,6.30317E-01,7.13888E-01,1.63418E+00,2.27404E+00,2.32490E+00,2.54158E+00,1.93257E+00,1.21870E+00
1.1687443e+00,1.36897E-04,1.40257E-04,6.60789E-02,8.80903E-02,5.89062E-01,6.72405E-01,1.55619E+00,2.18969E+00,2.27962E+00,2.54124E+00,2.00128E+00,1.31760E+00
1.3113526e+00,1.44324E-04,1.46966E-04,6.00055E-02,8.20456E-02,5.52154E-01,6.16030E-01,1.49249E+00,2.19424E+00,2.21043E+00,2.54159E+00,2.12010E+00,1.43742E+00
1.4713619e+00,1.68015E-04,1.71384E-04,5.19596E-02,7.09132E-02,5.09215E-01,5.74112E-01,1.41991E+00,2.08223E+00,2.12441E+00,2.61238E+00,2.16492E+00,1.54048E+00
1.6458952e+00,2.27963E-04,2.39622E-04,4.79900E-02,6.54341E-02,4.73257E-01,5.40976E-01,1.33151E+00,2.01013E+00,2.07436E+00,2.58005E+00,2.29152E+00,1.64716E+00
1.8473349e+00,3.58293E-04,3.69758E-04,4.17230E-02,5.90424E-02,4.44443E-01,5.01665E-01,1.26850E+00,1.94667E+00,2.01668E+00,2.54653E+00,2.31713E+00,1.71678E+00
2.0783539e+00,3.88647E-04,3.97657E-04,3.78674E-02,5.11876E-02,3.87839E-01,4.57702E-01,1.18353E+00,1.81169E+00,1.90491E+00,2.48066E+00,2.33735E+00,1.82047E+00

Figure 3 (Cont'd)

```
2.3319514e+00,2.92858E-04,3.00232E-04,3.38254E-02,4.53612E-02,3.60270E-01,4.20973E-01,1.10157E+00,1.72163E+00,1.79452E+00,2.43639E+00,2.36440E+00,1.88676E+00
2.6164926e+00,2.42762E-04,2.48953E-04,2.97013E-02,4.14414E-02,3.31870E-01,3.83927E-01,1.00838E+00,1.60198E+00,1.71395E+00,2.36585E+00,2.37278E+00,1.98122E+00
2.9357529e+00,2.81502E-04,2.86675E-04,2.74066E-02,3.63529E-02,2.99154E-01,3.49663E-01,9.44561E-01,1.49330E+00,1.57675E+00,2.24563E+00,2.31416E+00,1.92900E+00
3.2939689e+00,2.50758E-04,2.55674E-04,2.36309E-02,3.20902E-02,2.68206E-01,3.15867E-01,8.66372E-01,1.42190E+00,1.47301E+00,2.17336E+00,2.29103E+00,1.98153E+00
3.6958940e+00,2.68641E-04,2.73398E-04,2.22927E-02,3.03259E-02,2.39916E-01,2.82414E-01,7.63179E-01,1.31319E+00,1.34900E+00,2.00945E+00,2.15465E+00,1.88604E+00
4.1468612e+00,3.76944E-04,3.81236E-04,1.66724E-02,2.43178E-02,2.12834E-01,2.49871E-01,7.27865E-01,1.21951E+00,1.26348E+00,1.97658E+00,2.19209E+00,1.98578E+00
4.6528548e+00,5.81115E-04,5.85706E-04,1.56922E-02,2.11698E-02,1.86101E-01,2.19371E-01,6.61981E-01,1.14112E+00,1.21490E+00,1.95089E+00,2.24835E+00,2.11915E+00
5.2205890e+00,6.92461E-04,6.96598E-04,1.45833E-02,2.02583E-02,1.72348E-01,1.98123E-01,6.05369E-01,1.05138E+00,1.12365E+00,1.83871E+00,2.17276E+00,2.16005E+00
5.8575971e+00,8.23566E-04,8.28117E-04,1.37642E-02,1.77389E-02,1.66053E-01,1.85687E-01,5.65390E-01,1.00353E+00,1.04539E+00,1.74071E+00,2.13920E+00,2.12792E+00
6.5723321e+00,7.38765E-04,7.43104E-04,1.13185E-02,1.60374E-02,1.44353E-01,1.63748E-01,4.98918E-01,9.01183E-01,9.61153E-01,1.63226E+00,2.04881E+00,2.08017E+00
7.3742779e+00,6.29256E-04,6.33175E-04,1.01746E-02,1.29408E-02,1.22319E-01,1.46410E-01,4.53797E-01,7.85237E-01,8.41645E-01,1.50930E+00,1.93412E+00,2.08477E+00
8.2740759e+00,5.75969E-04,5.80215E-04,9.05895E-03,1.23095E-02,1.02990E-01,1.21892E-01,3.99952E-01,7.24141E-01,7.65343E-01,1.34986E+00,1.75683E+00,1.83804E+00
9.2836659e+00,5.77241E-04,5.82302E-04,7.17961E-03,9.24602E-03,9.73712E-02,1.07308E-01,3.44387E-01,6.17086E-01,6.77436E-01,1.23258E+00,1.64523E+00,1.78447E+00
1.0416445e+01,5.78455E-04,5.85291E-04,6.36695E-03,9.24639E-03,7.98967E-02,9.24053E-02,2.95713E-01,5.43214E-01,5.79082E-01,1.09493E+00,1.48828E+00,1.68068E+00
1.1687443e+01,5.98846E-04,6.09936E-04,7.18681E-03,9.82143E-03,7.85243E-02,9.73660E-02,2.94621E-01,5.32856E-01,5.77216E-01,1.07964E+00,1.46513E+00,1.63227E+00
1.3113526e+01,5.93506E-04,6.09289E-04,7.05998E-03,8.84060E-03,7.01140E-02,8.52170E-02,2.60141E-01,4.92751E-01,5.22922E-01,9.66173E-01,1.34303E+00,1.51734E+00
1.4713619e+01,5.74274E-04,5.92318E-04,6.64453E-03,1.01458E-02,6.67864E-02,8.51772E-02,2.51476E-01,4.51787E-01,4.74263E-01,9.09734E-01,1.25417E+00,1.40727E+00
1.6458952e+01,5.61524E-04,5.80343E-04,7.82933E-03,9.32081E-03,6.89575E-02,8.15814E-02,2.40749E-01,4.30326E-01,4.70638E-01,8.57628E-01,1.19825E+00,1.35671E+00
1.8473349e+01,5.53809E-04,5.71946E-04,7.19825E-03,8.99603E-03,6.35189E-02,7.66714E-02,2.20143E-01,3.94956E-01,4.17198E-01,7.82356E-01,1.08262E+00,1.27026E+00
2.0783539e+01,5.48552E-04,5.69557E-04,8.67801E-03,1.02328E-02,5.53739E-02,6.75728E-02,1.92238E-01,3.45412E-01,3.76366E-01,6.87744E-01,9.74137E-01,1.13427E+00
2.3319514e+01,5.51884E-04,5.79896E-04,8.40080E-03,9.74776E-03,5.26266E-02,6.27983E-02,1.80862E-01,3.09472E-01,3.25201E-01,6.13746E-01,8.48028E-01,1.03434E+00
```

Figure 4. Input file of the example discussed in **An Application of the Deconvolution Method**.

```
8,21
3,638.0,25.26
5,1234.0,35.13
7,1479.0,38.46
8,1524.0,39.04
9,1432.0,37.84
10,1015.0,31.86
11,716.0,26.76
12,433.0,20.81
4.46684E-07,7.05203E-06
3.98107E-06,7.14895E-06
3.98107E-05,6.28513E-06
0.000354813,5.67862E-06
0.003548134,6.37151E-06
0.035481339,6.35839E-06
0.177827941,2.19349E-06
0.316227766,2.46114E-06
0.562341325,2.05672E-06
0.707945784,2.30768E-06
0.891250938,2.06759E-06
1.258925412,6.18209E-06
2.238721139,6.50391E-06
2.818382931,2.05672E-06
3.548133892,2.31987E-06
5.011872336,1.83305E-06
6.309573445,1.0308E-06
7.943282347,1.16269E-06
11.22018454,1.08749E-06
12.58925412,9.69227E-07
14.12537545,0.0
```

Figure 5. Output File *.TBL of the example discussed in **An Application of the Deconvolution Method**.

```
Deconvolution Using the Maximum Entropy Algorithm

File with Input Data : DTKU9NU.CSV
File with Response Function: RF.CSV
Spectra is Stored in File : DTKU9NUR.OUT

Response Function in Units of: cm**2

Chi Square Using the Default Spectrum  = 2.62E+02
Final Chi Square (No Rebinning) =     8.00
Final Chi Square (Default Spectrum bins) =     8.10
Final Chi Square (Response Function bins) =     8.00

*** RESULTS FOR THE THE FINAL SPECTRUM: ***

NUM/DET   CTS/MEAS  CTS/CALC  (E-D)/S=  (E-D)/D=
       3  6.38E+02  6.27E+02    -0.443    -0.018
       5  1.23E+03  1.21E+03    -0.641    -0.018
       7  1.48E+03  1.49E+03     0.308     0.008
       8  1.52E+03  1.46E+03    -1.764    -0.045
       9  1.43E+03  1.43E+03     0.033     0.001
      10  1.02E+03  1.06E+03     1.437     0.045
      11  7.16E+02  7.05E+02    -0.407    -0.015
      12  4.33E+02  4.62E+02     1.400     0.067

*** RESULTS FOR THE DEFAULT SPECTRUM: ***

NUM/DET   CTS/MEAS  CTS/CALC  (E-D)/S=  (E-D)/D=
       3  6.38E+02  4.72E+02    -6.562    -0.260
       5  1.23E+03  9.85E+02    -7.098    -0.202
       7  1.48E+03  1.32E+03    -4.036    -0.105
       8  1.52E+03  1.39E+03    -3.495    -0.090
       9  1.43E+03  1.38E+03    -1.355    -0.036
      10  1.02E+03  1.17E+03     4.765     0.150
      11  7.16E+02  8.72E+02     5.813     0.217
      12  4.33E+02  6.21E+02     9.054     0.435

    NUM/DET  LAMBDA
       3     -6.211E-02
       5     -6.454E-02
       7      2.835E-02
       8     -1.600E-01
       9      3.097E-03
      10      1.598E-01
      11     -5.388E-02
      12      2.383E-01

Scaling Factor/Default Spectrum for best fit = 1.0740E+07
Scaling Factor/Default Spectrum used = 1.0740E+07

Default Spectrum Format: n Fluence Rate per Bin

Bin Structure Used : Response Function Bin Structure
```

Figure 6. Output File *.OUT of the example discussed in **An Application of the Deconvolution Method**.

Input File DTKU9NU.CSV

E (in MeV)	fl/t (I)	fl/t (F)	fl/t/L (I)	fl/t/L (F)	RB:Resp.F.	RB:Def.Sp.	(I)/(F)
4.6529E-07	3.9863E+00	4.6778E+00	3.4624E+01	4.0631E+01	4.0631E+01	4.7563E+01	8.5216E-01
5.2206E-07	3.9863E+00	4.9088E+00	3.4624E+01	4.2637E+01	4.2637E+01	4.7563E+01	8.1206E-01
5.8576E-07	3.9863E+00	5.0914E+00	3.4624E+01	4.4223E+01	4.4223E+01	4.7563E+01	7.8294E-01
6.5723E-07	3.9863E+00	5.2466E+00	3.4624E+01	4.5571E+01	4.5571E+01	4.7563E+01	7.5978E-01
7.3743E-07	3.9863E+00	5.3542E+00	3.4624E+01	4.6506E+01	4.6506E+01	4.7563E+01	7.4450E-01
8.2741E-07	3.9863E+00	5.4288E+00	3.4624E+01	4.7154E+01	4.7154E+01	4.7563E+01	7.3428E-01
9.2837E-07	3.9863E+00	5.5198E+00	3.4624E+01	4.7945E+01	4.7945E+01	4.7563E+01	7.2217E-01
1.0416E-06	3.9863E+00	5.5601E+00	3.4624E+01	4.8295E+01	4.8295E+01	4.7563E+01	7.1693E-01
1.1687E-06	3.9863E+00	5.6003E+00	3.4624E+01	4.8644E+01	4.8644E+01	4.7563E+01	7.1179E-01
1.3114E-06	3.9863E+00	5.6035E+00	3.4624E+01	4.8671E+01	4.8671E+01	4.7563E+01	7.1139E-01
1.4714E-06	3.8812E+00	5.4934E+00	3.4624E+01	4.9006E+01	4.9006E+01	4.7563E+01	7.0653E-01
1.6459E-06	3.9977E+00	5.6768E+00	3.4624E+01	4.9167E+01	4.9167E+01	4.7563E+01	7.0422E-01
1.8473E-06	4.0798E+00	5.8068E+00	3.4624E+01	4.9280E+01	4.9280E+01	4.7563E+01	7.0260E-01
2.0784E-06	3.9863E+00	5.6811E+00	3.4624E+01	4.9346E+01	4.9346E+01	4.7563E+01	7.0166E-01
2.3320E-06	3.9863E+00	5.6958E+00	3.4624E+01	4.9473E+01	4.9473E+01	4.7563E+01	6.9985E-01
2.6165E-06	3.9863E+00	5.6898E+00	3.4624E+01	4.9421E+01	4.9421E+01	4.7563E+01	7.0059E-01
2.9358E-06	3.9863E+00	5.7110E+00	3.4624E+01	4.9605E+01	4.9605E+01	4.7563E+01	6.9800E-01
3.2940E-06	3.9863E+00	5.7086E+00	3.4624E+01	4.9585E+01	4.9585E+01	4.7563E+01	6.9829E-01
3.6959E-06	3.9341E+00	5.5862E+00	3.4171E+01	4.8521E+01	4.8521E+01	4.7563E+01	7.0425E-01
4.1469E-06	3.8390E+00	5.5175E+00	3.3345E+01	4.7924E+01	4.7924E+01	4.6938E+01	6.9579E-01
4.6529E-06	3.8390E+00	5.4991E+00	3.3345E+01	4.7764E+01	4.7764E+01	4.6938E+01	6.9811E-01
5.2206E-06	3.8390E+00	5.4871E+00	3.3345E+01	4.7660E+01	4.7660E+01	4.6938E+01	6.9964E-01
5.8576E-06	3.8390E+00	5.4759E+00	3.3345E+01	4.7563E+01	4.7563E+01	4.6938E+01	7.0106E-01
6.5723E-06	3.8390E+00	5.4758E+00	3.3345E+01	4.7562E+01	4.7562E+01	4.6938E+01	7.0109E-01
7.3743E-06	3.8390E+00	5.4566E+00	3.3345E+01	4.7395E+01	4.7395E+01	4.6938E+01	7.0355E-01
8.2741E-06	3.8390E+00	5.4719E+00	3.3345E+01	4.7528E+01	4.7528E+01	4.6938E+01	7.0158E-01
9.2837E-06	3.8390E+00	5.4708E+00	3.3345E+01	4.7519E+01	4.7519E+01	4.6938E+01	7.0172E-01
1.0416E-05	3.8390E+00	5.4534E+00	3.3345E+01	4.7367E+01	4.7367E+01	4.6938E+01	7.0396E-01
1.1687E-05	3.8390E+00	5.4378E+00	3.3345E+01	4.7233E+01	4.7233E+01	4.6938E+01	7.0598E-01
1.3114E-05	3.8390E+00	5.4152E+00	3.3345E+01	4.7036E+01	4.7036E+01	4.6938E+01	7.0893E-01
1.4714E-05	3.7378E+00	5.2519E+00	3.3345E+01	4.6852E+01	4.6852E+01	4.6938E+01	7.1172E-01
1.6459E-05	3.8500E+00	5.4059E+00	3.3345E+01	4.6821E+01	4.6821E+01	4.6938E+01	7.1218E-01
1.8473E-05	3.9291E+00	5.5232E+00	3.3345E+01	4.6873E+01	4.6873E+01	4.6938E+01	7.1139E-01
2.0784E-05	3.8390E+00	5.3771E+00	3.3345E+01	4.6705E+01	4.6705E+01	4.6938E+01	7.1395E-01
2.3320E-05	3.8390E+00	5.3628E+00	3.3345E+01	4.6581E+01	4.6581E+01	4.6938E+01	7.1586E-01
2.6165E-05	3.8390E+00	5.1781E+00	3.3345E+01	4.4976E+01	4.4976E+01	4.6938E+01	7.4139E-01
2.9358E-05	3.8390E+00	5.3167E+00	3.3345E+01	4.6180E+01	4.6180E+01	4.6938E+01	7.2206E-01
3.2940E-05	3.8390E+00	5.3455E+00	3.3345E+01	4.6431E+01	4.6431E+01	4.6938E+01	7.1817E-01
3.6959E-05	3.7375E+00	5.1567E+00	3.2464E+01	4.4791E+01	4.4791E+01	4.6938E+01	7.2479E-01
4.1469E-05	3.5528E+00	4.9168E+00	3.0859E+01	4.2707E+01	4.2707E+01	4.1027E+01	7.2258E-01
4.6529E-05	3.5528E+00	4.8931E+00	3.0859E+01	4.2501E+01	4.2501E+01	4.1027E+01	7.2607E-01
5.2206E-05	3.5528E+00	4.7868E+00	3.0859E+01	4.1578E+01	4.1578E+01	4.1027E+01	7.4220E-01
5.8576E-05	3.5528E+00	4.7406E+00	3.0859E+01	4.1176E+01	4.1176E+01	4.1027E+01	7.4943E-01
6.5723E-05	3.5528E+00	4.7804E+00	3.0859E+01	4.1522E+01	4.1522E+01	4.1027E+01	7.4319E-01
7.3743E-05	3.5528E+00	4.8459E+00	3.0859E+01	4.2091E+01	4.2091E+01	4.1027E+01	7.3315E-01
8.2741E-05	3.5528E+00	4.5525E+00	3.0859E+01	3.9542E+01	3.9542E+01	4.1027E+01	7.8040E-01
9.2837E-05	3.5528E+00	4.7687E+00	3.0859E+01	4.1421E+01	4.1421E+01	4.1027E+01	7.4501E-01
1.0416E-04	3.5528E+00	4.7635E+00	3.0859E+01	4.1375E+01	4.1375E+01	4.1027E+01	7.4583E-01
1.1687E-04	3.5528E+00	4.6699E+00	3.0859E+01	4.0563E+01	4.0563E+01	4.1027E+01	7.6077E-01
1.3114E-04	3.5528E+00	4.7550E+00	3.0859E+01	4.1301E+01	4.1301E+01	4.1027E+01	7.4716E-01
1.4714E-04	3.4592E+00	4.5958E+00	3.0859E+01	4.0999E+01	4.0999E+01	4.1027E+01	7.5268E-01
1.6459E-04	3.5629E+00	4.7440E+00	3.0859E+01	4.1088E+01	4.1088E+01	4.1027E+01	7.5104E-01
1.8473E-04	3.6362E+00	4.8204E+00	3.0859E+01	4.0909E+01	4.0909E+01	4.1027E+01	7.5433E-01
2.0784E-04	3.5528E+00	4.6727E+00	3.0859E+01	4.0587E+01	4.0587E+01	4.1027E+01	7.6032E-01
2.3320E-04	3.5528E+00	4.6944E+00	3.0859E+01	4.0775E+01	4.0775E+01	4.1027E+01	7.5680E-01
2.6165E-04	3.5528E+00	4.6411E+00	3.0859E+01	4.0312E+01	4.0312E+01	4.1027E+01	7.6549E-01
2.9358E-04	3.5528E+00	4.6771E+00	3.0859E+01	4.0625E+01	4.0625E+01	4.1027E+01	7.5961E-01
3.2940E-04	3.3744E+00	4.4266E+00	2.9309E+01	3.8449E+01	3.8449E+01	4.1027E+01	7.6230E-01

Figure 6 (Cont'd)

3.6959E-04	3.0494E+00	3.9154E+00	2.6487E+01	3.4009E+01	3.4009E+01	3.3773E+01	7.7882E-01
4.1469E-04	3.0494E+00	3.9435E+00	2.6487E+01	3.4253E+01	3.4253E+01	3.3773E+01	7.7328E-01
4.6529E-04	3.0494E+00	3.9621E+00	2.6487E+01	3.4415E+01	3.4415E+01	3.3773E+01	7.6964E-01
5.2206E-04	3.0494E+00	3.9619E+00	2.6487E+01	3.4413E+01	3.4413E+01	3.3773E+01	7.6969E-01
5.8576E-04	3.0494E+00	3.9456E+00	2.6487E+01	3.4271E+01	3.4271E+01	3.3773E+01	7.7287E-01
6.5723E-04	3.0494E+00	3.9198E+00	2.6487E+01	3.4047E+01	3.4047E+01	3.3773E+01	7.7796E-01
7.3743E-04	3.0494E+00	3.9014E+00	2.6487E+01	3.3887E+01	3.3887E+01	3.3773E+01	7.8162E-01
8.2741E-04	3.0494E+00	3.9091E+00	2.6487E+01	3.3954E+01	3.3954E+01	3.3773E+01	7.8009E-01
9.2837E-04	3.0494E+00	3.9166E+00	2.6487E+01	3.4019E+01	3.4019E+01	3.3773E+01	7.7858E-01
1.0416E-03	3.0494E+00	3.8787E+00	2.6487E+01	3.3690E+01	3.3690E+01	3.3773E+01	7.8620E-01
1.1687E-03	3.0494E+00	3.8773E+00	2.6487E+01	3.3678E+01	3.3678E+01	3.3773E+01	7.8648E-01
1.3114E-03	3.0494E+00	3.8672E+00	2.6487E+01	3.3590E+01	3.3590E+01	3.3773E+01	7.8853E-01
1.4714E-03	2.9691E+00	3.7666E+00	2.6487E+01	3.3602E+01	3.3602E+01	3.3773E+01	7.8825E-01
1.6459E-03	3.0582E+00	3.8618E+00	2.6487E+01	3.3447E+01	3.3447E+01	3.3773E+01	7.9190E-01
1.8473E-03	3.1210E+00	3.9416E+00	2.6487E+01	3.3451E+01	3.3451E+01	3.3773E+01	7.9182E-01
2.0784E-03	3.0494E+00	3.8120E+00	2.6487E+01	3.3111E+01	3.3111E+01	3.3773E+01	7.9994E-01
2.3320E-03	3.0494E+00	3.8249E+00	2.6487E+01	3.3223E+01	3.3223E+01	3.3773E+01	7.9725E-01
2.6165E-03	3.0494E+00	3.8137E+00	2.6487E+01	3.3125E+01	3.3125E+01	3.3773E+01	7.9960E-01
2.9358E-03	3.0494E+00	3.7937E+00	2.6487E+01	3.2951E+01	3.2951E+01	3.3773E+01	8.0382E-01
3.2940E-03	3.1813E+00	3.9516E+00	2.7632E+01	3.4323E+01	3.4323E+01	3.3773E+01	8.0507E-01
3.6959E-03	3.4215E+00	4.2156E+00	2.9719E+01	3.6616E+01	3.6616E+01	3.6117E+01	8.1162E-01
4.1469E-03	3.4215E+00	4.2267E+00	2.9719E+01	3.6713E+01	3.6713E+01	3.6117E+01	8.0949E-01
4.6529E-03	3.4215E+00	4.2193E+00	2.9719E+01	3.6649E+01	3.6649E+01	3.6117E+01	8.1091E-01
5.2206E-03	3.4215E+00	4.1898E+00	2.9719E+01	3.6392E+01	3.6392E+01	3.6117E+01	8.1663E-01
5.8576E-03	3.4215E+00	4.1814E+00	2.9719E+01	3.6319E+01	3.6319E+01	3.6117E+01	8.1827E-01
6.5723E-03	3.4215E+00	4.1831E+00	2.9719E+01	3.6334E+01	3.6334E+01	3.6117E+01	8.1794E-01
7.3743E-03	3.4215E+00	4.1873E+00	2.9719E+01	3.6370E+01	3.6370E+01	3.6117E+01	8.1712E-01
8.2741E-03	3.4215E+00	4.1531E+00	2.9719E+01	3.6074E+01	3.6074E+01	3.6117E+01	8.2384E-01
9.2837E-03	3.4215E+00	4.1396E+00	2.9719E+01	3.5957E+01	3.5957E+01	3.6117E+01	8.2652E-01
1.0416E-02	3.4215E+00	4.1344E+00	2.9719E+01	3.5911E+01	3.5911E+01	3.6117E+01	8.2756E-01
1.1687E-02	3.4215E+00	4.1255E+00	2.9719E+01	3.5833E+01	3.5833E+01	3.6117E+01	8.2936E-01
1.3114E-02	3.4215E+00	4.1134E+00	2.9719E+01	3.5728E+01	3.5728E+01	3.6117E+01	8.3180E-01
1.4714E-02	3.3314E+00	3.9879E+00	2.9719E+01	3.5576E+01	3.5576E+01	3.6117E+01	8.3537E-01
1.6459E-02	3.4313E+00	4.1008E+00	2.9719E+01	3.5517E+01	3.5517E+01	3.6117E+01	8.3675E-01
1.8473E-02	3.5018E+00	4.1939E+00	2.9719E+01	3.5592E+01	3.5592E+01	3.6117E+01	8.3499E-01
2.0784E-02	3.4215E+00	4.0659E+00	2.9719E+01	3.5316E+01	3.5316E+01	3.6117E+01	8.4151E-01
2.3320E-02	3.4215E+00	4.0512E+00	2.9719E+01	3.5189E+01	3.5189E+01	3.6117E+01	8.4456E-01
2.6165E-02	3.4215E+00	4.0528E+00	2.9719E+01	3.5202E+01	3.5202E+01	3.6117E+01	8.4422E-01
2.9358E-02	3.4215E+00	4.0360E+00	2.9719E+01	3.5056E+01	3.5056E+01	3.6117E+01	8.4774E-01
3.2940E-02	3.9376E+00	4.6052E+00	3.4202E+01	4.0000E+01	4.0000E+01	3.6117E+01	8.5504E-01
3.6959E-02	4.8778E+00	5.6882E+00	4.2368E+01	4.9407E+01	4.9407E+01	4.8040E+01	8.5753E-01
4.1469E-02	4.8778E+00	5.6997E+00	4.2368E+01	4.9507E+01	4.9507E+01	4.8040E+01	8.5579E-01
4.6529E-02	4.8778E+00	5.6796E+00	4.2368E+01	4.9332E+01	4.9332E+01	4.8040E+01	8.5883E-01
5.2206E-02	4.8778E+00	5.6276E+00	4.2368E+01	4.8881E+01	4.8881E+01	4.8040E+01	8.6676E-01
5.8576E-02	4.8778E+00	5.6076E+00	4.2368E+01	4.8707E+01	4.8707E+01	4.8040E+01	8.6986E-01
6.5723E-02	4.8778E+00	5.5691E+00	4.2368E+01	4.8373E+01	4.8373E+01	4.8040E+01	8.7586E-01
7.3743E-02	4.8778E+00	5.5760E+00	4.2368E+01	4.8432E+01	4.8432E+01	4.8040E+01	8.7478E-01
8.2741E-02	4.8778E+00	5.5680E+00	4.2368E+01	4.8363E+01	4.8363E+01	4.8040E+01	8.7604E-01
9.2837E-02	4.8778E+00	5.5251E+00	4.2368E+01	4.7990E+01	4.7990E+01	4.8040E+01	8.8284E-01
1.0416E-01	4.8778E+00	5.4534E+00	4.2368E+01	4.7368E+01	4.7368E+01	4.8040E+01	8.9445E-01
1.1687E-01	4.8778E+00	5.4652E+00	4.2368E+01	4.7471E+01	4.7471E+01	4.8040E+01	8.9251E-01
1.3114E-01	4.8778E+00	5.4102E+00	4.2368E+01	4.6993E+01	4.6993E+01	4.8040E+01	9.0159E-01
1.4714E-01	4.7493E+00	5.2027E+00	4.2368E+01	4.6413E+01	4.6413E+01	4.8040E+01	9.1285E-01
1.6459E-01	4.8368E+00	5.2296E+00	4.1892E+01	4.5294E+01	4.5294E+01	4.8040E+01	9.2489E-01
1.8473E-01	4.8222E+00	5.1740E+00	4.0925E+01	4.3910E+01	4.3910E+01	4.3267E+01	9.3202E-01
2.0784E-01	4.7116E+00	4.9843E+00	4.0925E+01	4.3293E+01	4.3293E+01	4.3267E+01	9.4529E-01
2.3320E-01	4.7116E+00	4.9559E+00	4.0925E+01	4.3047E+01	4.3047E+01	4.3267E+01	9.5070E-01
2.6165E-01	4.7116E+00	4.8708E+00	4.0925E+01	4.2307E+01	4.2307E+01	4.3267E+01	9.6733E-01
2.9358E-01	4.9154E+00	5.0386E+00	4.2694E+01	4.3765E+01	4.3765E+01	4.3267E+01	9.7553E-01
3.2940E-01	5.2865E+00	5.2966E+00	4.5918E+01	4.6006E+01	4.6006E+01	4.7410E+01	9.9809E-01
3.6959E-01	5.2865E+00	5.2247E+00	4.5918E+01	4.5381E+01	4.5381E+01	4.7410E+01	1.0118E+00
4.1469E-01	5.2865E+00	5.0962E+00	4.5918E+01	4.4265E+01	4.4265E+01	4.7410E+01	1.0374E+00

Figure 6 (Cont'd)

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4.6529E-01  5.2865E+00  4.9612E+00  4.5918E+01  4.3092E+01  4.3092E+01  4.7410E+01  1.0656E+00
5.2206E-01  7.3271E+00  6.7128E+00  6.3643E+01  5.8307E+01  5.8307E+01  4.7410E+01  1.0915E+00
5.8576E-01  1.1045E+01  9.8785E+00  9.5932E+01  8.5803E+01  8.5803E+01  8.6390E+01  1.1180E+00
6.5723E-01  1.1522E+01  1.0013E+01  1.0008E+02  8.6976E+01  8.6976E+01  8.6390E+01  1.1507E+00
7.3743E-01  1.2392E+01  1.0428E+01  1.0764E+02  9.0573E+01  9.0573E+01  8.2925E+01  1.1884E+00
8.2741E-01  1.0624E+01  8.6667E+00  9.2277E+01  7.5278E+01  7.5278E+01  8.2925E+01  1.2258E+00
9.2837E-01  7.4020E+00  5.8406E+00  6.4293E+01  5.0731E+01  5.0731E+01  5.3739E+01  1.2673E+00
1.0416E+00  7.4020E+00  5.6453E+00  6.4293E+01  4.9034E+01  4.9034E+01  5.3739E+01  1.3112E+00
1.1687E+00  9.4848E+00  7.0748E+00  8.2384E+01  6.1451E+01  6.1451E+01  5.3739E+01  1.3406E+00
1.3114E+00  1.3279E+01  9.4001E+00  1.1534E+02  8.1648E+01  8.1648E+01  8.6168E+01  1.4127E+00
1.4714E+00  1.2929E+01  8.9275E+00  1.1534E+02  7.9642E+01  7.9642E+01  8.6168E+01  1.4482E+00
1.6459E+00  1.3317E+01  9.0107E+00  1.1534E+02  7.8042E+01  7.8042E+01  8.6168E+01  1.4779E+00
1.8473E+00  1.3591E+01  8.8722E+00  1.1534E+02  7.5295E+01  7.5295E+01  8.6168E+01  1.5319E+00
2.0784E+00  2.0951E+01  1.3392E+01  1.8197E+02  1.1632E+02  1.1632E+02  8.6168E+01  1.5644E+00
2.3320E+00  3.4926E+01  2.1684E+01  3.0336E+02  1.8835E+02  1.8835E+02  1.6632E+02  1.6107E+00
2.6165E+00  2.6463E+01  1.6613E+01  2.2985E+02  1.4430E+02  1.4430E+02  1.6632E+02  1.5929E+00
2.9358E+00  1.1045E+01  6.8417E+00  9.5932E+01  5.9426E+01  5.9426E+01  5.7517E+01  1.6143E+00
3.2940E+00  1.0074E+01  6.4022E+00  8.7499E+01  5.5609E+01  5.5609E+01  5.7517E+01  1.5735E+00
3.6959E+00  8.3051E+00  5.1103E+00  7.2137E+01  4.4387E+01  4.4387E+01  4.4109E+01  1.6252E+00
4.1469E+00  8.3051E+00  4.9249E+00  7.2137E+01  4.2777E+01  4.2777E+01  4.4109E+01  1.6864E+00
4.6529E+00  8.8503E+00  5.1995E+00  7.6873E+01  4.5162E+01  4.5162E+01  4.4109E+01  1.7022E+00
5.2206E+00  9.8435E+00  5.8690E+00  8.5499E+01  5.0978E+01  5.0978E+01  4.7198E+01  1.6772E+00
5.8576E+00  8.3167E+00  4.9988E+00  7.2238E+01  4.3419E+01  4.3419E+01  4.7198E+01  1.6637E+00
6.5723E+00  5.5354E+00  3.3071E+00  4.8080E+01  2.8725E+01  2.8725E+01  2.8348E+01  1.6738E+00
7.3743E+00  5.0488E+00  3.2202E+00  4.3854E+01  2.7971E+01  2.7971E+01  2.8348E+01  1.5678E+00
8.2741E+00  4.1624E+00  2.6808E+00  3.6154E+01  2.3286E+01  2.3286E+01  2.8959E+01  1.5527E+00
9.2837E+00  4.1624E+00  2.7541E+00  3.6154E+01  2.3922E+01  2.3922E+01  2.8959E+01  1.5114E+00
1.0416E+01  6.8265E+00  4.5671E+00  5.9294E+01  3.9669E+01  3.9669E+01  2.8959E+01  1.4947E+00
1.1687E+01  1.1230E+01  7.7658E+00  9.7538E+01  6.7453E+01  6.7453E+01  6.7453E+01  1.4460E+00
0.1311E+02
```

APPENDIX A. THE PRINCIPLE OF MAXIMUM ENTROPY

The entropy of the distribution f(x), in the form given in Skilling (1989), is given by

$$S = - \int \{f(x) \ln (f(x)/f(x)^{DEF}) + f(x)^{DEF} - f(x)\} \, dx, \qquad (A.1)$$

where $f(x)^{DEF}$ is the default distribution. This is a generalization of the usual cross-entropy formula

$$S_{CE} = - \int \{f(x) \ln (f(x)/f(x)^{DEF})\} \, dx, \qquad (A.2)$$

that appears as the first term in (A.1). The second term in (A.1) is an additive constant that ensures that the global maximum of S (at $f(x) = f(x)^{DEF}$) is zero, a sometimes convenient but not essential requirement. The third term in (A.1) ensures that in the absence of any other constraints, when S is maximized, $f(x) = f(x)^{DEF}$ (rather than just being proportional).

Before we go on, a note on terminology: some authors use the term entropy when referring to what we call here cross-entropy, while others use the term cross-entropy for the negative of S_{CE}, and, as a result, end up with the principle of *minimum* cross-entropy. Other names for S_{CE} (or its negative) are relative entropy, direct divergence, expected weight of evidence, Kullback-Leibler distance, and Kullback number. To keep the arguments as close to the standard literature as possible, when discussing the maximum entropy principle in this section, we will be referring to maximization of S_{CE}, not of S. For an axiomatic derivation of S (rather than S_{CE}) we refer the reader to Skilling (1989).

The maximum entropy principle provides a rule for determining a positive, additive distribution given definite but incomplete constraints. The probability distribution p(x) of a variable x is an example of a positive, additive distribution. It is positive by construction, and it is additive in the sense that the overall probability in a domain equals the sum of the probabilities in any decomposition into subdomains. In our case, the positive, additive distribution to be determined is the neutron energy spectrum, f(x), and the constraints are the measurements and the experimental errors associated with them. The maximum entropy principle states that from all distributions f(x) satisfying the set of constraints, one should choose the one for which S_{CE} is

maximal. A parallel procedure is used in statistical mechanics, where given constraints on the total energy and number of particles of a gas, entropy maximization is used to infer a distribution that is uniform in space and Maxwellian in velocity. There are several approaches that lead to the maximum entropy principle. We summarize two of them below.

In the original derivation by Jaynes (1957, 1968), the use of the maximum entropy principle is justified on the basis of the cross-entropy's unique properties as an uncertainty measure. Arguments originating in information theory show that the Shannon entropy,

$$S_{SH} = - \sum f_i \ln (f_i / f_i^{DEF}) \qquad (A.3)$$

is the appropriate measure of the amount of uncertainty expressed in a distribution. These arguments follow from requiring three consistency conditions that are generally accepted as requirements for an uncertainty measure. Expression (A.3) for the case $f_i^{DEF} = 1$ was originally derived by Shannon (1948) for discrete distributions. Jaynes (1968) showed that (A.2) can be derived by generalizing to continuous distributions, in which case $f(x)^{DEF}$ appears as an "invariant measure" function (as well as playing the role of default distribution). Once a measure of the amount of uncertainty is established, it then seems reasonable to choose from all distributions f(x) satisfying a set of constraints the one for which S_{CE} is maximal, since this is the distribution that "assumes the least" about the unknown f(x). While agreeing with what is known, it will be maximally non-committal with regard to all information except the data, and, therefore, the most unbiased estimate.

An independent justification that makes no reference to information theory is given by Shore and Johnson (1980). Jaynes (1957) had already remarked that inferences made using any other information measure than the entropy may lead to contradictions. Shore and Johnson (1980) consider the consequences of requiring that methods of inference be self-consistent. They introduce four axioms that are all based on one fundamental principle: if a problem can be solved in more than one way, the results should be consistent. Informally, they state their axioms as follows:

I. *Uniqueness*: The result should be unique,

II. *Invariance*: The choice of coordinate system should not matter,

III. *System independence*: It should not matter whether one accounts for independent information about independent systems separately in terms of different densities or together in terms of a joint density, and

IV. *Subset independence*: It should not matter whether one treats an independent subset of system states in terms of a separate different conditional density or in terms of the full system density. They go on to show that given information in the form of a set of constraints on expected values, there is only one distribution satisfying the set of constraints that can be chosen by a procedure that satisfies the four axioms, and this unique distribution can be obtained by maximizing S_{CE}. Therefore, if a method of inference is based on a variational principle, maximizing any function but S_{CE} will lead to inconsistencies unless that function and entropy have identical maxima (any monotonic function of the entropy will work, for example).

APPENDIX B. A USER'S GUIDE TO THE CODE MAXED

INTRODUCTION

MAXED is a FORTRAN code that applies the maximum entropy principle to the deconvolution of multisphere neutron spectrometer data. This guide describes the code in sufficient detail to allow a user to prepare the necessary input files and to run it on a PC under DOS. While most of the information needed to run the code is read from input data files, some information is provided by the user at run time.

While developing the code, we added a subroutine that carries out a second, separate deconvolution of the data using the SAND-II algorithm [see McElroy et al. (1967)]. This was done to allow us to compare the results of our code with the results obtained using the SAND-II algorithm. While we favor using the maximum entropy method (for reasons given in the main body of the text), we have decided to make the SAND-II option available. More information on the SAND-II option is given below.

A package with a copy of the code and other related files can be obtained from the authors by sending an e-mail request to mreg@eml.doe.gov or goldhagn@eml.doe.gov. This package includes the following files:

1. MAXED.FOR, the source code;

2. MAXED.EXE, an executable code;

3. RF.CSV, the response function file used for the example discussed in **An Application of the Deconvolution Method**;

4. INPUT.CSV, the input file used for the example discussed in **An Application of the Deconvolution Method**; and

5. LF90.ERR, the run time error message file.

MAXED.EXE is an executable code for use under DOS. To run the program, the response function file, the input file, and the run time error message file must be in the same directory as the executable file.

DESCRIPTION OF THE INPUT DATA FILES

Running MAXED requires the user to create two files: one with information on the response functions of the detectors (the response functions file), the other one with information about the measurements and the default spectrum (the input file). These files are described below. The format of the numbers that appear in these files (e.g., real or integer numbers) are indicated in parenthesis, after a brief description of the quantity under consideration. Real numbers may be written in either exponential notation (e.g., 6.385E+02) or in standard notation (e.g., 638.5). Both files are comma delimited (the same format as spreadsheet programs' *.CSV files).

An example of a response function file is shown in Figure 3. The response function file has the following structure:

1. One line with the number of detectors (integer number).

2. One line with the number of energy bin edges (integer number).

3. One line with the units of the response function (10 characters or less).

4. One line with the lowest energy bin edge of the response functions, in MeV (real number).

5. Several lines with the upper energy bin edge of the response functions, in MeV (real number), followed by all the detectors' response function values for that bin (real numbers). The response functions for all the detectors must have the same energy bin structure.

When the response function file is read in, each detector is assigned a detector number in the following way. When the rows with the detectors' response function values are read in, the first number that is read in (column 1) is the upper energy bin edge of the response functions. Then, the first response function value that is read in (column 2) is assigned to detector 1, the second response function value (column 3) to detector 2, the third response function value (column 4) to detector 3, etc.

An example of an input file is shown in Figure 4. The input file has the following structure:

1. One line with the number of measurements (integer number), followed by the number of energy bin edges used for the default spectrum (integer number).

2. Several lines with information on the measurements, each line with a detector number (integer number), the measured data for that detector, usually in counts per second (real

number), and the error assigned to that measurement, in the same units as the measured data (real number).

3. Several lines with information on the default spectrum, each line with the value of the lower energy bin edge in MeV (real number), followed by the value of the default spectrum for that bin (real number).

4. One line with the highest energy bin edge of the default spectrum, in MeV (real number), followed by a zero.

We believe that the ability to incorporate the measurement errors in the deconvolution (many deconvolution methods do not take errors into account) is one of the very useful features of the code MAXED. However, we caution the user that it is important to assign realistic values to these experimental errors. The errors assigned should take into account not only statistical fluctuations in the number of counts, but also estimates of the contribution from other sources. If the errors assigned to the measurements are too small, the solution spectrum will show an excessive amount of fine structure. This can be easily checked by inspection, after which the problem can be run again using better estimates of the errors. In this way, extraneous fine structure can be avoided without applying smoothing.

RUNNING THE PROGRAM MAXED

To run the program, type "MAXED" at the DOS prompt. The following information is entered by the user in response to prompting questions from the program:

1. The names of the input, output, and response function files;

2. The form of the default spectrum (e.g., differential energy spectrum, lethargy spectrum or fluence rate per bin).

In addition, the following options are available:

1. A choice of energy bin structure;

2. A choice of parameters for scaling the default spectrum;

3. A choice of parameters for the simulated annealing subroutine; and

4. The choice of carrying out a second, separate deconvolution of the data using the SAND-II algorithm.

Names of input, output, and response function files

The input and response function file names can be any legal DOS file name (up to eight characters for the file name, followed by a period, followed by up to three characters for the extension). The output file name is limited to eight characters in length (with no extension).

Form of the default spectrum

The default spectrum which appears in the input file can be in one of three formats in:

1. {neutron fluence rate per bin}/{width of bin in E(MeV)}, which is the discretized version of the differential neutron fluence rate per unit energy, $d\Phi/dE$;

2. {neutron fluence rate per bin}/{width of bin in ln(E(MeV))}, which is the discretized version of the differential neutron fluence rate per unit lethargy, $Ed\Phi/dE = d\Phi/d(\ln E)$; and

3. {neutron fluence rate per bin}.

Bin structure for the deconvolution

The code MAXED provides the user with four options for the bin structure of the deconvolution:

1. a fine bin structure;

2. four bins per decade;

3. the bin structure of the default spectrum; and

4. the bin structure of the response functions.

If option (1) is chosen, the energy bin structure used for the deconvolution is generated by overlapping the bin structures of both the default spectrum and the response functions, so that there is a bin edge whenever either the default spectrum or the response functions have a bin edge. If option (2) is chosen, the energy bin structure used is of equal width in the logarithm of the energy (measured in MeV), and it has four bins per decade. If option (3) is chosen, the

energy bin structure used is the same as that of the default spectrum. If option (4) is chosen, the energy bin structure used is the same as that of the response functions.

The default spectrum and the response functions are rebinned to the final energy bin structure whenever necessary, and the rebinning is done using linear interpolation in the logarithm of the energy. In all cases, the lowest bin edge used for the deconvolution is the maximum of the lowest bin edges of the default spectrum and the response functions, and the highest bin edge used is the minimum of the highest bin edges of the default spectrum and the response functions.

No matter what energy bin structure is chosen, in all cases the output is provided in three forms: using the chosen bin structure, and rebinned after deconvolution of the data to the bin structure of the default spectrum, and to the bin structure of the response functions. We recommend using option (1), and rebinning to either the bin structure of the default spectrum or to the bin structure of the response functions. In general, the χ^2 of the rebinned spectrum will be larger than the χ^2 of the deconvolution that is done using option (1). In most cases, this is acceptable. However, if rebinning results in an increase in χ^2 that is unacceptably large, this can be an indication that there is an excessive amount of fine structure in the deconvoluted spectrum. This is usually due to having assigned unrealistically small errors to the measurements.

Scale factor for the default spectrum

While a suitable estimate of the shape of the default spectrum is often available, the normalization (absolute magnitude) that corresponds to the conditions of the measurement is usually unknown. Given the default spectrum, the code MAXED calculates the multiplicative scale factor that provides the best fit to the data (that is, it calculates the multiplicative factor that minimizes the chi-square of the default spectrum). The user is then given one of three options:

1. To scale the default spectrum, using the multiplicative scale factor calculated by MAXED;

2. Not to scale the default spectrum; and

3. To scale the default spectrum, using a scale factor entered by the user.

We recommend option (1) for most cases. If the default spectrum is the result of a reliable calculation that is normalized to the conditions of the measurement, then option (2) might be a better choice.

Parameters for the simulated annealing subroutine

The simulated annealing subroutine requires the user to enter two parameters:

1. The temperature, T; and

2. The temperature reduction factor, RT.

Here, temperature does not refer to an actual physical temperature, but is just the name of a parameter that regulates the algorithm. In most cases, an initial value for the temperature can be chosen as follows. Note that simulated annealing first builds up a rough view of the surface by moving with large step lengths at a high temperature. In our case, the function is convex in the large, and has only one maximum at a large enough scale length. Therefore, *choose an initial temperature that is high enough so that no new maxima are found in the first set of iterations at constant temperature*. This can be determined by observing the value in the "NEW MAXIMA THIS TEMPERATURE" line, which is printed to screen under the heading "INTERMEDIATE RESULTS BEFORE NEXT TEMPERATURE REDUCTION" as the simulated annealing subroutine is running. If this value is zero for the first set of iterations, the temperature that was chosen should be adequate. For most of our applications, a temperature of 1000 has been large enough to satisfy this requirement. This procedure corresponds to initially probing the surface to within a scale length that is so large that we already are at the global maximum for the given scale length. Then, as the temperature is decreased, the algorithm is given the chance to inspect the "fine wrinkles" of the function at temperatures that are still high enough to escape from any of the local maxima. After searching through the local maxima, the algorithm should arrive at the global maximum.

RT must satisfy $0 < RT < 1$. We recommend setting the temperature reduction factor to 0.85, as suggested in Corana et al. (1987) and in Goffe et al. (1994).

The SAND-II option

The SAND-II algorithm is an iterative process that alters an initial trial spectrum so as to obtain a solution spectrum that agrees better with the measured data. Here, we set the initial trial spectrum equal to the default spectrum used for the maximum entropy deconvolution.

The user is required to enter the maximum number of iterations. In addition, the user has the choice of using one of two convergence criteria:

1. A maximum fractional deviation in any bin; and

2. The value of the chi-square.

If option (1) is chosen, the SAND-II deconvolution is considered finished when either the fractional deviation in each bin from one iteration to the next falls below the value entered, or when the maximum number of iterations is reached. If option (2) is chosen, the SAND-II deconvolution is considered finished when either the chi-squared becomes equal to or less than the number of measurements, or when the maximum number of iterations is reached.

DESCRIPTION OF THE OUTPUT DATA FILES

The output consists of some or all of the following files:

1. OUTPUT.OUT;
2. OUTPUT.TBL;
3. OUTPUT.SII; and
4. OUTPUT.TS2.

Here "OUTPUT" is the output file name set by the user. The first two files are always created, and the last two files are only created if the user chooses the SAND-II option while running the code.

The files OUTPUT.TBL and OUTPUT.TS2 contain information on the results of the deconvolution using the maximum entropy method and the SAND-II algorithm. Figure 5 shows an examples of an OUTPUT.TBL file.

The files OUTPUT.OUT and OUTPUT.SII contain the results of the deconvolution. The first row lists the name of the input file used in the deconvolution. The second row labels the columns. For each of the rows that follow, the columns list the following quantities:

1. The value of the lower energy bin edge, in MeV;

2. The value of the default spectrum for that bin, in {neutron fluence rate per bin};

3. The value of the solution spectrum for that bin, in {neutron fluence rate per bin};

4. The value of the default spectrum for that bin, in {neutron fluence rate per bin}/{width of bin in ln(E(MeV))}, which is the discretized version of the differential neutron fluence rate per unit lethargy, $E d\Phi/dE = d\Phi/d(\ln E)$.

5. The value of the solution spectrum for that bin, in {neutron fluence rate per bin}/{width of bin in ln(E(MeV))};

6. The value of the solution spectrum for that bin, in {neutron fluence rate per bin}/{width of bin in ln(E(MeV))}, rebinned to the response function bin structure;

7. The value of the solution spectrum for that bin, in {neutron fluence rate per bin}/{width of bin in ln(E(MeV))}, rebinned to the default spectrum bin structure; and

8. The ratio of default spectrum to final spectrum for that bin (column 2 divided by column 3).

   Finally, the last row lists the value of the highest energy bin edge, in MeV.

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