WPO 39830

date: December 6, 1993

to: Distribution

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subject: WIPP Anhydrite Fracture Modeling

Introduction

We have recently performed some analytical linear elastic fracture mechanics (LEFM) calculations that predict the development of long, of the order of kilometers, thin, on the order of millimeters thick, dynamic fractures in response to waste gas generation at WIPP. It is not clear at this time whether a gas-driven hydrofracture can be considered a design feature for WIPP (in the sense that the cracking process may act as a "safety valve" to limit gas pressures) or a design problem for WIPP (in the sense that cracking may enhance flow and transport of materials away from the repository). The essence of our model, which assumes the growth of a horizontal, circular, axisymmetric crack, is outlined briefly below. Our analytical model is not very detailed. Indeed, it can be argued that LEFM may be an overly conservative approach for gas-driven hydrofracture at WIPP. If hydrofracture of the anhydrite is to be considered a WIPP design feature, and if simple LEFM calculations are deemed not sufficiently accurate, then we believe a that more defensible conceptual and numerical modeling of the WIPP anhydrite fractures should be pursued. For example, more detailed models considering dynamic, three-dimensional, nonsymmetric, coupled fracture and two-phase flow analyses are all possible. However, the ability to develop these models and associated parameter data base relative to the WIPP formation soon enough to meet WIPP program needs has yet to be determined. If compliance with WIPP regulations precludes large gas-driven hydrofractures or if WIPP schedules and resources precludes the development of more advanced gas-driven hydrofracture models, then the WIPP program may be faced with the need to enact some form of engineering alternative, such as build in gas storage or waste modification to limit the production of gas.

Because of the potential importance of fracturing of the WIPP anhydrites, we are committed to continue to explore theoretical and conceptual models searching for modeling solutions that will be applicable to WIPP anhydrites. Never-the-less, depending on schedule and resource allocation, there is a risk to the WIPP program in depending on being able to accurately and defensibly predict the location and added gas storage volume of gas-driven hydrofracture for a December 1996 compliance target.

SWCF-A: CO/CCA, 1.1.7.1 QA

Background

Linear elastic fracture mechanics (LEFM) is the simplest and most classical of the theories of fracture. It assumes that the medium follows Hooke's Law and that the fracture process zone is of negligible size compared to other dimensions in the problem. Both of these assumptions seem reasonable when applied to hydrofracture problems in geologic media. Indeed, LEFM is the most widely accepted model for hydrofracture in the gas and oil industry. However, any other fracture model requires complex constitutive models and the solution of nonlinear equations, and thus becomes computationally difficult. Quite simply, LEFM is to cracks as Timoshenko Beam Theory is to Beams.

While complex continuum mechanics finite element models have been used in the past, it is now known that unless very special consideration of fracture mechanics is taken into account, these models cannot predict fractures objectively. This lack of objectivity with respect to cracking was discovered in the late 1970's and continues to be the focus of intense research in the engineering mechanics field even today. It is fair to say that currently no consensus exists among research engineers regarding an appropriate method for finite element modeling of cracks.

In what follows, we outline the development of an axisymmetric LEFM model for gas-driven hydrofracture at WIPP. Using representative parameters in the model, a large crack is predicted. Then, we show that this crack may grow dynamically. Finally, we examine the effects of markerbed dip.

Analytical Model for Gas-Driven, Circular, Horizontal Hydrofracture

Consider the case of a circular crack in an infinite elastic medium. This assumption is appropriate when considering a crack at WIPP where the crack length is much less than the 650 meter depth of the crack below the land surface. This type of analysis is reasonable for crack lengths at the WIPP of less than 300 meters. For cracks longer than 300 meters the assumption of an infinite medium is conservative because by ignoring the free surface an overprediction of the crack length and an underprediction of the gas storage volume results.

Table 1 documents all of the input parameters in our model of a gas-driven hydrofracture. Table 2 documents all of the calculated responses from the gas-driven hydrofracture model.

Table 1-Input Parameters to Gas-Driven Hydrofracture Model

Parameter	Units	Definition	Comments
a_0	meters	Initial crack Radius	predicted crack lengths are not sensitive to this parameter
E	MPa	Young's Modulus	31,000 for halite
ν	none	Poisson's Ratio	0.25 (use halite rather than anhydrite parameters)
ϕ	none	Porosity	markerbed porosity (halite porosity assumed 0)
K_{1c}	$MPa\sqrt{m}$	Critical Stress Intensity Factor	(fracture toughness) assumed range for anhydrite (5 - 0) in situ values unknown
σ_0	MPa	Overburden Stress	14.8
V_{room}	m^3	Gas Accessible Void of Repository at Maximum Compaction	45000 (note 120 rooms for the repository) 375 m³/room.
n	moles	Quantity of Gas	function of time total potential 1650 moles/drum
R	$(m^3$ MPa)/ (moles K)	Gas Constant	8.23 x 10^{-6}
T	Deg K	Absolute Temperature	assumed 300 for WIPP

Table 2-Calculated Responses in the Model

Variable	Units	Definition	Comments
a	meters	Crack Length	radius of the circular crack
V_{crack}	m^3	Gas Accessible Storage Volume in the Crack	V_{salt}, $V_{markerbed}$ assumed 0
P	MPa	Gas Pressure	P less than or equal to P_{crit}
P_{crit}	MPa	Critical Gas Pressure	pressure required to extend crack
Δ	meters	Crack Opening Displacement	elliptical shape in profile
K_I	$MPa\sqrt{m}$	Stress Intensity Factor	less than or equal to K_{Ic}

For an embedded, penny shaped crack in an infinite elastic medium, subject to a remote compression σ_0 and internal pressure P, a solution can be found (using superposition of solutions) in "The Stress Analysis of Cracks Handbook," by Tada, Paris, and Irwin, Del Publishing, 1985:

$$K_I=\frac{2}{\pi}(P-\sigma_0)\sqrt{\pi a} \tag{1}$$

For this work P will be determined from the ideal gas law. To determine the evolution of P, the amount of gas, n, as a function of time is required. We have assumed that for the first 550 years gas is generated at the rate of 2 moles/drum/year and that from 550 through 1100 years gas is generated at the rate of 1 mole/drum/year. After 1100 years it is assumed that the waste is totally degraded and no further gas generation is possible. The repository is assumed to contain 120 rooms with each room containing 6800 drums of waste.

Crack opening displacement, Δ, can be found from the same reference:

$$\Delta=\left(\frac{2}{\pi}\right)\frac{4(1-\nu^2)}{E}(P-\sigma_0)\sqrt{a^2-r^2} \qquad (2)$$

Assuming that the pressure, P, in the system can never exceed the pressure that causes crack growth, P_{crit}, we can rewrite equation 1 as shown in equation 3 below. (In other words, if the pressure exceeds the critical pressure, P_{crit}, a crack would grow and thus create enough void volume to reduce the pressure, P, to P_{crit}.) Note that at incipient crack growth $K_I=K_{Ic}$, and $P=P_{crit}$, leading to:

$$P_{crit}=\frac{K_{Ic}}{2}\sqrt{\frac{\pi}{a}}+\sigma_0 \qquad (3)$$

The volume of the crack can be determined by using the crack opening displacement given in equation 2 via:

$$V=\int_{r=0}^{r=a}\int_{\theta=0}^{\theta=2\pi}\Delta(r d\theta)dr$$

The integration results in:

$$V_{crack}=\frac{16(1-\nu^2)(P-\sigma_0)a^3}{3E} \qquad (4)$$

Recalling the ideal gas law PV=nRT, note that V must include all the gas assessable void volume, i.e., volume of the room, V_{room}, volume of the crack, V_{crack}, and volume available in the marker beds, $V_{markerbed}$. Here we are assuming that the gas storage volume in the halite, V_{salt}, is insignificant. So we may write:

$$P_{crit}(V_{room}+V_{crack}+V_{Marker\ Bed})=nRT$$

Inserting the crack volume from equation 4 into the equation above and rearranging gives:

$$V_{room}+\frac{16(1-\nu^2)(P_{crit}-\sigma_0)a^3}{3E}+V_{Marker\ Bed}=\frac{nRT}{P_{crit}} \tag{5}$$

The void volume in the markerbed can be defined in terms of the markerbed porosity, crack length and markerbed thickness as follows:

$$V_{Marker\ Bed}=\pi a^2(\phi_{Marker\ Bed})(t_{Marker\ Bed})$$

and finally we can write the following transcendental equation in a. (Note that P_{crit} is a function of 'a' defined in equation 3)

$$a=\left[\frac{3E}{16(1-\nu^2)(P_{crit}-\sigma_0)}\left(\frac{nRT}{P_{crit}}-V_{room}-V_{Marker\ Bed}\right)\right]^{1/3} \tag{6}$$

These equations can and have been solved numerically for crack length, a, and the other responses listed in Table 2. Examples of the solutions are shown in Figure 1.

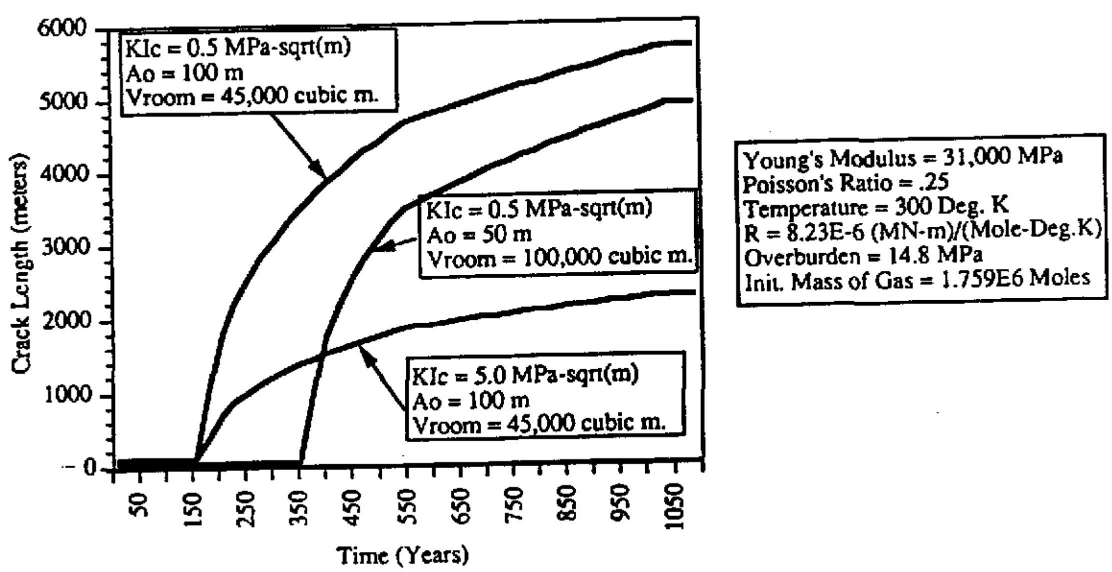

Figure 1.

Other variations using this approach have been investigated;

however, now we examine the possibility of dynamic crack behavior.

Unstable Equilibrium

For this discussion we will consider the potential gas storage volume in the markerbed, $V_{markerbed}$, to be zero. It simplifies the equations slightly and otherwise has no impact on the discussion.

Now consider equation 5. Rewrite the equation to solve for the number of moles of gas, n, as a function of crack length. Specifically the equation determines the crack length, a, resulting in an equilibrium condition for a given amount, n, of gas in the system.

$$n=\frac{1}{RT}\,(P_{crit})\,\left(\frac{16(1-\nu^2)(P_{crit}-\sigma_0)\;a^3}{3E}+V_{room}\right)$$

P_{crit} can be eliminated using equation 3:

$$n=\frac{1}{RT}\,\left(\sigma_0+\frac{K_{Ic}}{2}\sqrt{\frac{\pi}{a}}\right)\,\left(\frac{16(1-\nu^2)\left(\frac{K_{Ic}}{2}\sqrt{\frac{\pi}{a}}\right)\;a^3}{3E}+V_{room}\right) \tag{7}$$

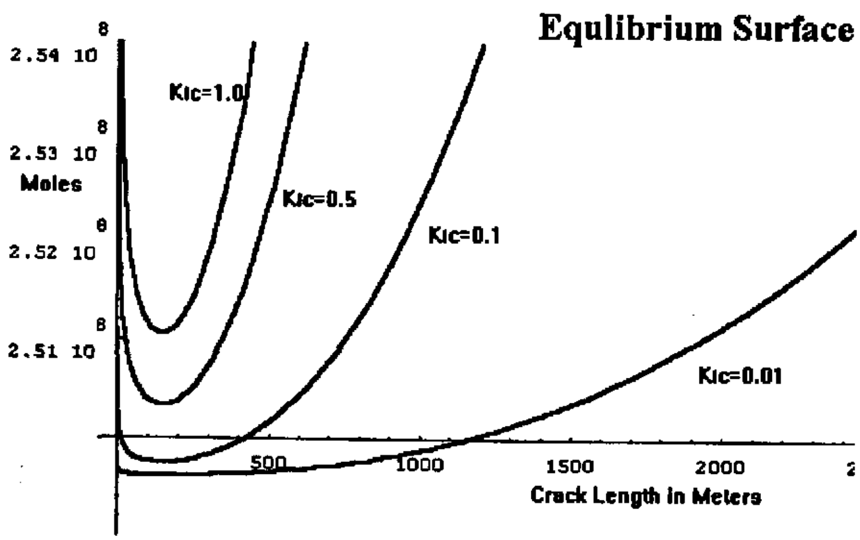

Figure 2

Plotting this equation with several different values for K_{Ic} results in the family of curves shown in Figure 2.

These results indicate that for a given amount of gas there may be two equilibrium states. One state results in a short, more pressurized crack, and the other state results in a much longer, and less pressurized, crack. Assume K_{Ic}=0.5 and an initial crack length, a_0, 50 meters long. As the gas builds up it will first reach the equilibrium state on the left side of the curve.

Any subsequent generation of gas will result in two new equilibrium conditions, one less than 50 meters and one much longer in length, roughly 300 meters (see Figure 2). Since there is no way for the crack to grow shorter, it will attempt to transition to the other, much longer crack. The transition between these two equilibrium states is likely to be dynamic.

Because the sudden transition from shorter cracks to longer cracks is quite sensitive to both the initial crack length, a_0, and to the local value of K_{Ic}, which is expected to be quite heterogenous, it is not likely that we will be able to model crack growth, either crack length or crack storage volume, within 300 meters of the repository with any confidence at all. As we extend beyond 300 meters the equilibrium crack length is monotonically increasing, indicating a stable crack growth region.

Effects of Markerbed Dip

Consider the case where the markerbeds dip at a small constant slope, s. The sloping markerbed can be represented as a boundary stress gradient as shown in the left hand of Figure 3.

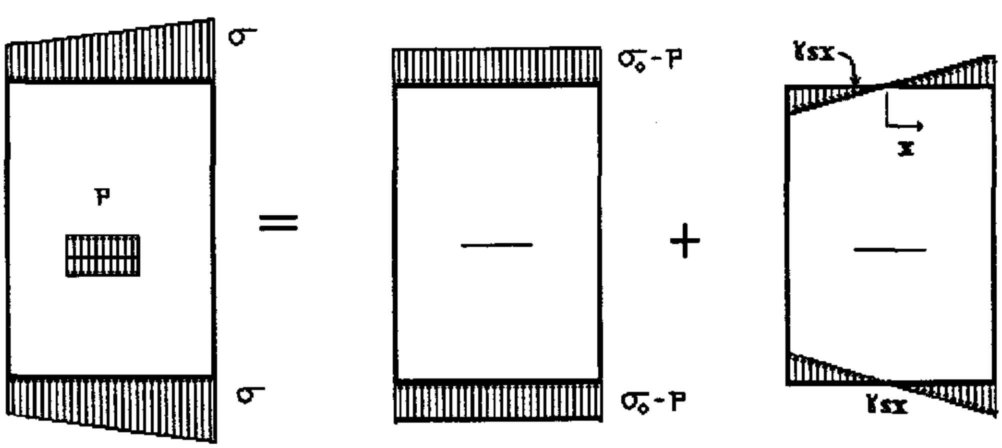

Figure 3

By invoking linear superposition of stress fields, Figure 3 shows that this problem can be solved by using the solution from the pressurized embedded circular crack and adding to it the solution for an embedded circular crack with a linear external pressure gradient. (Again we refer to "The Stress Analysis of Cracks Handbook," by Tada, Paris, and Irwin, Del Publishing)

Thus the stress intensity factor can be written as:

$$K_I = [2(P-\sigma_0)+\frac{4\gamma sx}{3}]\ \sqrt{\frac{a}{\pi}} \tag{8}$$

Where gamma is the pressure change due to the dip per unit length of along the crack. Making the assumption of equilibrium crack growth, Equation 8 may be rearranged to determine the internal pressure required to propagate the crack.

$$P_{crit} = \frac{1}{2}\ [K_{Ic}\sqrt{\frac{\pi}{a}}-\frac{4\gamma sx}{3}] + \sigma_0 \tag{9}$$

Consider the following conditions:

P_{lith} = 14.8 MPa - at the center of the circular crack
K_{Ic} = 0.3 MPa - (meters)$^{0.5}$
Depth = 655 meters
s = slope = 0.015 (Here we assume a dip of 1.5%. Markerbed 139 average slope is expected to be 2-4%).
a = 150 meters (crack length)

If we look at the critical pressure at three positions around the circular crack, directly down dip, normal to the dip, and directly up dip, we see that even this very slight slope has a significant impact on fracture behavior. Down dip the critical pressure is 14.86 MPa; normal to the dip the critical pressure is 14.82 MPa; and up dip the critical pressure is 14.79 MPa. At the very least these results tell us that the crack will tend to grow up dip in a noncircular fashion.

Let's look further at the critical pressure required to propagate a crack in the up dip direction. Figure 4, shows the pressure relative to lithostatic required to propagate the crack as a function of crack length. Note that the pressure required to keep the crack open near the center of the circular crack, i.e., near the repository, must be above P_{lith}. It appears that the pressure, P_{crit}, necessary to propagate the crack is less than the pressure necessary to keep the crack open. This means the possibility must be considered that crack growth will occur in dynamic spurts even though the gas generation rates are very slow in the repository.

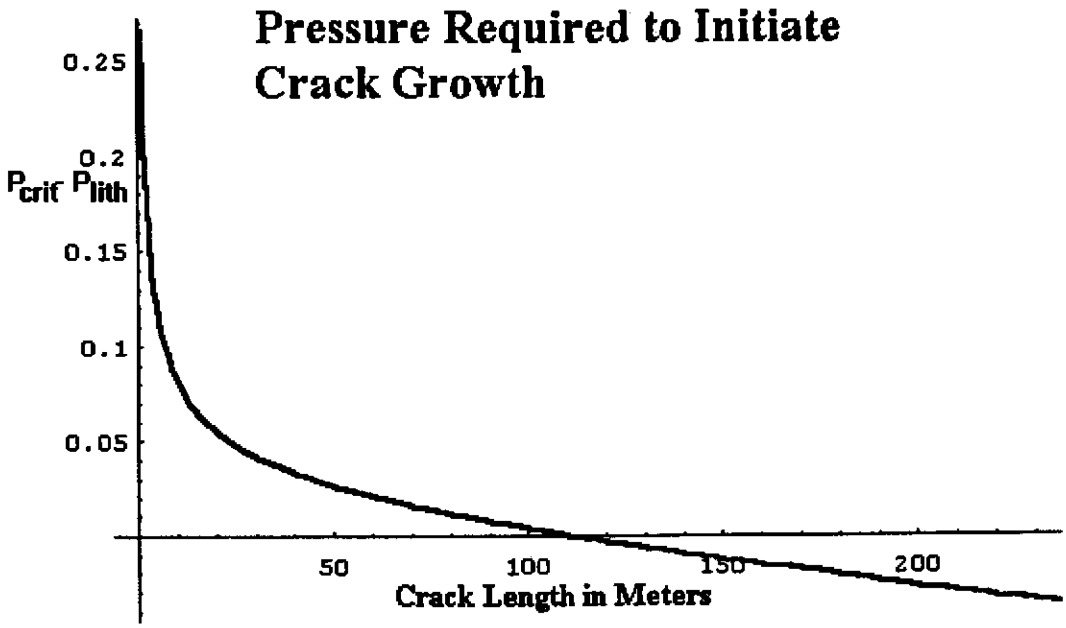

Figure 4

Conclusions

Assuming WIPP expected gas generation potentials, and assuming simple linear elastic fracture mechanics (LEFM) we have predicted that a horizontal circular crack kilometers in radius is possible at WIPP. We have shown that dynamic growth may occur and needs further consideration. Furthermore, we have demonstrated that crack shape and growth directions are sensitive to slight variations in slope, 1.5%, in either the markerbed or equivalently in the surface topology. However, we feel that it is possible to carefully account for these concerns with the LEFM approach, as well as the expected heterogeneous nature of the anhydrite. This suggests that using LEFM theory may lead to conservative and defensible crack growth predictions in the WIPP anhydrites.

Furthermore, based on this work we believe that an advanced model of gas-driven hydrofracture, if necessary for WIPP compliance calculation, would need to simulate at least some aspects of nonlinear, dynamic, three-dimensional, nonsymmetric, coupled fracture-flow behavior. This last approach would require technology development. However, to be useful for WIPP, such technology needs to be developed in time to meet our program milestones. The authors wish to investigate the potential for developing this capability as well as the parameter data base of material properties and geometric variability needed as input to the model. However, we feel compelled to state that on the current expected WIPP program schedule and resource allocation there is some risk that a more advanced gas driven hydrofracture model may not be achievable. The WIPP program must be prepared with a fall back position, such as engineered alternatives, if LEFM models are not acceptable and/or if a more advanced gas-driven hydrofracture model is not achieved.

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Fracture Model : ~~64~~ NQ : Salado Flow

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